Energy in Contemporary Textbooks

1. The most common concept of energy is presented in the form: energy cannot be created nor destroyed but only transformed.
2. In some textbooks energy is defined as the capacity of doing work.
3. Some authors say that we do not know what energy is.

In what follows, definitions or concepts of energy in textbooks are chronologically presented.

In 1903 Voigt published *Thermodynamics*. Energy is understood here as the capacity of a body to do work[1]. The core of Voigt's account resides in the mechanical equivalent of heat. Over several paragraphs, he deals with different ways of determining the mechanical equivalent of heat: from specific heats, as Mayer had done; by friction, as Joule did; due to the compression and rarefaction of gases, Joule 1845; through the electric current, Joule 1843; by impact, as Hirn had determined, etc. (p. 80-94).

Preston, in *Theory of Heat*, 1919, tells us that the ether would have been postulated to serve as a vehicle for energy because matter would not have been sufficient to account for the energy that passed through space[2]. This was going to be used to explain potential energy[3]: phenomena such as the pendulum or throwing a body into the air are seen as interchanges of energy between bodies and the ether. [4]

According to Preston, thermodynamics is based on two principles, the first of which is called the principle of equivalence. This principle, which would have been discovered by Joule, states the proportionality between heat and work[5].

In 1926 the 11th edition of *Physics* of Müller and Pouillet appeared. The crucial term in the domain studied is equivalence. The subject is presented with the title "principle of equivalence". The first law of thermodynamics is formulated in the following terms: heat and work are equivalent[6]. Beyond a small account, with special attention to Joule, 1850, and to Hirn's experiment, 1858, a list of mechanical equivalents is presented: values, methods of determining and authors. The first determining of mechanical equivalent is attributed to Joule, 43, and the first calculation to Mayer, 42. The principle of conservation of energy is introduced with reservations.

The introduction of this principle is justified through the finality of physics of giving a global image of unity[7]. With the statement, energy is indestructible, the principle would not be an empirical law, but a postulate, said the authors.[8] The postulate is conforming to experience by virtue of the law of equivalence.

In sum, what matters from the experimental point of view is the principle of equivalence. To an indestructible energy there is no reference, here, it is a matter of a postulate.

Saha and Srisvastava, 1935, in *Treatise on Heat*, presented the principle as a result of human experience, the impossibility of a machine of perpetual motion.[9] The principle could also be deduced from the mechanical point of view of the universe, but the authors judged it as not very clear[10].

In Wolf's *Elements of Physics*, 1949, the first law of thermodynamics is formally given by the equation: the variation of internal energy is equal to the sum of the variation of heat and of the variation of forms which are not heat[11]. One way of expressing the law is given in the form, a *perpetuum mobile* is impossible[12]. Hund, in *Theoretical Physics*, 1956, began by taking heat as a substance. [13] In the second step, he showed that there are phenomena which are not comprehensible by heat-substance[14]. Here, he established the idea of heat-motion and heat as a form of energy.[15]

Allen and Maxwell, in the compendium on heat, 1962, presented a historical section, according to which, Jouleestablished the principle of equivalence between heat and work in 1843, which, at the time when the authors wrote, was called the first law of thermodynamics.[16] Furthermore, the principle of equivalence is explicit through the constancy of the mechanical equivalent of heat[17]. The establishment of the equivalent by experience would justify the following step, the introduction of hypothesis, heat as a form of energy[18].

The Nobel Laureate, Richard Feynman, said in his *Lectures* in the sixties, "It is important to realize that in physics today we have no knowledge of what energy *is*" (Vol. I, 4-1).

In 25/26th edition of *Physics* of Westphal, 1970, energy is conceived of as a reserve of work[19]. Energy would be the capacity of doing work, but would be distinguished from work by describing a state whereas work would be an event.[20]

Hudson and Nelson said in 1982, that energy could neither be created nor destroyed, but only transformed. This proposition, known as the law of conservation of energy, is based on experiment[21]. The authors added that energy was not a substance but a quantity.[22]

For Keller, Gettys and Skove, in *Physics* of 1993, work and heat are energies transferred between the system and the environment.[23] The authors began the chapter on the conservation of energy with three quotations: of which, there was one of Poincaré – we do not know how to give a definition of energy, the principle simply says that there is something that remains constant – and one of Feynman– it is important to realise that we do not know what energy is.[24].

Cutnell and Johnson in 4th edition of *Physics*, 1997, presented the principle of conservation of energy in the form, energy can neither be created nor destroyed, but can only be converted from one form to another[25]. Heat is defined as "energy which flows"[26].

In the 11th edition of *Experimental Physics* of Bergmann and Schaefer, 1998, it can be read "*no one knows what energy really is*"[27]. Energy and work are distinguished in the following way: work is spoken of if energy is transported; energy is used if it is not in motion or it is linked to a moving body[28]. For the authors, the frequently used expression 'Forms of energy' is misleading. The different "types" of energy had an origin in different forms as it is transported (work, heat, radiation) or in different links to matter (kinetic, potential, chemical electrical energy, etc.)[29].

In 1999, Breithaupt defined energy as the capacity of doing work[30]. Energy exists in different forms, continued the author, being able to be transformed from one form to another[31]. Further to this, the author added, energy can be transferred from one body to another by two methods, work and heat being defined as energy transferred by each one of these methods[32].

According to Tipler, in *Physics*, edition of 2000, the concept of energy describes the capacity of doing work[33]. Energy can appear in different forms, such as kinetic or potential[34]. Heat is understood as energy which passes from one body to another[35].

Dransfeld, in the 9th edition of *Physics*, of 2001 began by announcing that we do not know how to answer the question, what is energy[36]. As experiments have shown, energy cannot be produced nor destroyed, but remains constant in a closed system[37]. In the processes which occur in nature, one form of energy is transformed into another. As forms of energy, gravitational energy, the energy of heat, kinetic and electric energy, amongst others are indicated[38].

In the *Thermodynamics* of Çengel and Boles, of 2002, thermodynamics is defined as the science of energy[39]. Nevertheless, the authors signalled the difficulty in defining energy, which they said could be considered as the capacity to cause changes[40]. The principle of conservation of energy was formulated in the following way, energy can change from one form to another, but the total quantity remains constant. Heat is defined as a form of energy that is transferred between two systems due to a difference of temperature[41]; and work as transference of energy associated with force[42]. The authors linked work and heat with processes and energy with state[43]. Thus it is said that systems possess energy, but not heat or work[44].

In the *Fundamentals of Physics* of Halliday, Resnick and Walker, 2003, the difficulty in defining energy is signalled[45] . The work is understood as energy which is transported and the meaning of 'transport' is insinuated or is similar to an electronic bank transfer[46].

In the 11th edition of *Physics* of *Sears and Zemansky*, of Young and Freedman, of 2004, the law of conservation of energy is given by the equation, the sum of the variations of kinetic, potential and internal energy is constant[47]. The equation is read in the form, energy is never created or destroyed, but only changes its form[48].

Anderson, in his *Thermodynamics*, 2005, writes: "Everyone knows what energy is, but it is an elusive topic if you are looking for a deep understanding" (p. 33).

[1] "Entzieht man dem Körper die an ihm aufgewandte und in der ursprünglichen Form verschwundene Arbeit oder Wärme nicht in Form von Wärme oder Arbeit, so kommt sie eben der *Arbeitsfähigkeit*, der Energie des Körpers zu gute" (p. 78).

[2] "Energy is again often stated to be only associated with matter, so that matter has been defined as the vehicle of energy; this, however, does not hold according to our limitation of the word matter, for we know that energy in immense quantities is perpetually passing through space with the velocity of light [...]

The ether, then, is the great vehicle of energy; and, indeed, it is chiefly on this account that the ether has been postulated" (p. 80).

[3] "It may not be out of place to examine here the meaning of the term potential energy" (p. 87).

[4] "An evident reply to the question of what becomes of the motion of a projectile rising upwards is that it passes into the ether. [...] The oscillation from kinetic to potential, and from potential to kinetic, in the case of the pendulum is then, from this point of view, merely an interchange of energy of motion going on between the mass of the pendulum and the ether around it. [...] The oscillation of energy, then, is from ether to matter, and from matter to ether, and on this oscillation all the physical life of the universe depends" (p. 88).
[5] "The modern science of thermodynamics is based on two fundamental principles, both of which relate to the conversion of heat into work. The first of these is the principle of equivalence established by Joule, and is represented algebraically by the equation
$W = JH$.
This principle, which is known as the *first law of thermodynamics*, asserts than when work is spent in producing heat, the quantity of work spent is directly proportional to the quantity of heat generated [...] This conception is derived from the dynamical theory, according to which heat is regarded as a form of energy" (p. 667).

[6] "Die am engsten an die unmittelbare Erfahrung sich anschließende Formulierung des ersten Hauptsatzes, die von jeder Hypothese, etwa über die Natur der Wärme frei ist, besagt daher einfach: Wärme und mechanische Arbeit sind äquivalent" (p. 109).

[7] "Da nun gerade der Gewinn eines einheitlichen, einfachen Gesamtbildes das Hauptziel der physikalischen Wissenschaft bildet, ist man gezwungen, über das formale, rein beschreibende Äquivalenzprinzip hinauszugehen, wenn auch bei diesem Schritte der Boden der unmittelbaren Erfahrung verlassen werden muß" (p. 126).

[8] "*Energie (beliebiger Form) kann weder erzeugt, noch vernichtet werden*. Die einfachste Gestalt nimmt das Energieprinzip wohl in der Form an: *Die Summe aller einem abgeschlossenen System innewohnenden Energieformen bleibt bei sämtlichen Umwandlungen desselben konstant*.
Bei noch etwas schärferer Fassung nimmt der vorangehende Gedankengang folgende Gestalt an: Energie wird als unzerstörbar angesehen. Das Energieprinzip ist somit zunächst kein empirisches Gesetz, sondern ein *Postulat*, das sich allerdings mit den Erfahrungstatsachen (Äquivalenzgesetz) durchaus im Einklang befindet" (p. 126).
[9] "It follows from a result of human experience which may be stated in this form:-
"*It is impossible to design a machine which will create energy out of nothing and produce perpetual motion. Energy can only be transformed from one form to the other.*"" (p. 434).

[10] "It may also be deduced from the mechanical view of the Universe which treats all energy as kinetic or potential. This view is however open to criticism and is not quite clear" (p. 434).

[11] "U2-U1=Q+A. Dies soll heißen, daß der Unterschied der inneren Energie nach der Energiezufuhr von außen, U2, gegenüber dem Wert U1 von vorher gerade gleich der in den Körper neu hineingesteckten Energie sein muß. Diese wird dabei noch aufgespalten dargestellt in Form einer eventuellen Wärmemenge Q und irgendeiner sonstigen Energie A der eben charakterisierten Art, die nicht Wärme ist […] Diese Gleichung ist **der erste Hauptsatz der Thermodinamik** in quantitativer Form" (p. 272).

[12] "Man hat für derartige Phantasiemaschinen den Namen "*Perpetuum mobile*" geprägt. Daher wird schließlich der erste Hauptsatz auch gern in der kurzen Form ausgesprochen: *Ein Perpetuum mobile ist nicht möglich*" (p. 273).

[13] "Wärme ist also (zunächst) eine Substanz wie Materiemenge" (p. 28).

[14] "Es gibt nun Erscheinungen, die mit der Vorstellung der Wärme als Substanz nicht erfaßt werden: die Erzeugung von Wärme durch den elektrischen Strom, die Entstehung von Wärme bei Reibung […]" (p. 49).

[15] "Die genauere Untersuchung dieser Erscheinungen führte zu der Ansicht, daß Wärme eine Energieform sei. Zunächst führte sie zu der Ansicht, Wärme sei Bewegung der kleinsten Teile der Körper und damit eine Form von Bewegungsenergie (*F. Mohr* 1837); später zu der Ansicht, Wärme sei eine Form der Energie […] (*R. Mayer*1842)" (p. 49-50).

[16] "The experiments completed by Joulein 1843 firmly established the principle of equivalence between heat and work, a principle which is now generally termed the first law of thermodynamics" (p. 23).

[17] "J. P. Joulewas the first to establish on a satisfactory experimental basis the **Principle of Equivalence**. This principle may be expressed by saying that when exchange occurs between work and heat, the *ratio* of the exchange is fixed" (p. 284).

[18] "When such an exact relation between mechanical energy and heat has been established by experiment, it is not difficult to take a further step by introducing the hypothesis that heat is itself a form of energy, so that in the exchanges considered there is a transmutation from one form of energy to another" (p. 285).
[19] „In einem Körper, an dem Verschiebungs- oder Beschleunigungsarbeit verrichtet wurde, ist also ein vom Betrage dieser Arbeit abhängiger Vorrat an Arbeit aufgespeichert. Man nennt ihn die Energie des Körpers, und diese ist – als gespeicherte Arbeit – mit dieser gleichartig" (p. 38).

[20] “Da Energie Arbeitsfähigkeit, also latente, aufgespeicherte Arbeit ist, so messen wir sie in den gleichen Einheiten wie die Arbeit [...] Man muß indessen zwischen Energie und Arbeit begrifflich unterscheiden. Energie beschreibt einen Zustand, Arbeit ist ein zeitlich ablaufender Vorgang“ (p. 39).

[21] “Thus, energy can only be changed from one form to another; it cannot be created or destroyed. This conclusion, based on experiment, is known as the **law of conservation of energy**” (p. 95).

[22] “Energy is an extremely important concept in physics. Although it appears in many different forms, it is not a physical substance, but a calculated quantity” (p. 95).

[23] „***Heat*** is energy transferred between a system and its environment because of a temperature difference between them [...] ***Work*** is energy transferred between a system and its environment by means independent of the temperature difference between them“ (p. 423).

[24] "As we cannot give a general definition of energy, the principle of the conservation of energy simply signifies that there is something which remains constant. Well, whatever new notions of the world future experiments may give us, we know beforehand that there will be something which remains constant and which we shall be able to call energy.
Henry Poincare [...]
It is important to realize that in physics today, we have no knowledge of what energy is.
Richard P. Feynman" (p. 194).

[25] „**The principle of conservation of energy**
Energy can neither be created nor destroyed, but can only be converted from one form to another” (p. 177).

[26] “**Definition of Heat**
Heat is energy that flows from a higher-temperature object to a lower-temperature object because of the difference in temperatures” (p. 359).

[27] "**Definition der Energie**. Dabei stoßen wir gleich auf eine Schwierigkeit: *Niemand weiß, was Energie wirklich ist*. Der Physiker befindet sich dabei fast im selben Dilemma wie der Laie. [...] Er [der Energiesatz] besagt, daß sich bei keinem bisher in der Natur beobachteten Vorgang die Gesamtenergie eines abgeschlossenen Systems verändert hat" (p. 135).

[28] "Der Begriff der Energie ist keineswegs nur ein anderes Wort für Arbeit. Erst wenn Energie in ganz bestimmter Weise transportiert wird, sprechen wir von Arbeit. Wenn sie dagegen ruht oder mit einem bewegten Körper fest verbunden ist, nennen wir sie schlicht Energie" (p. 136).

[29] "Es gibt im Grunde nur eine wohldefinierte Größe Energie, die in einem Körper oder in einem Raumbereich vorhanden ist. Verschiedene "Erscheinungsformen" oder "Arten" dieser Energie beruhen entweder auf der unterschiedlichen Art und Weise, wie sie transportiert wird (Arbeit, Wärme,

Strahlung) oder auf den verschiedenen Möglichkeiten, wie sie mit Materie verbunden sein kann (kinetische, potentielle, chemische, elektrische usw. Energie). Es ist daher etwas irreführend, von verschiedenen *Energieformen* zu sprechen, wie es oft geschieht. Denn es handelt sich immer nur um verschiedene *Erscheinungsformen* ein und derselben Größe *Energie*" (p. 136).
[30] "**Energy is defined as the capacity to do work**" (p. 157).

[31] "**Energy exists in different forms**"
"**Energy can be transformed from any one form into other forms**. Whenever energy changes from one form into other forms, the total amount of energy is unchanged. This general rule is known as the **principle of conservation of energy**" (p. 157).

[32] „Energy is the capacity of a body to do work. Energy can be transferred from one body to another by two methods:
1. Work is energy transferred by means of a force moving its point of application.
2. Heat is energy transferred by means other than a force. A temperature difference is said to exist between two bodies if heat transfer between the two bodies could occur" (p. 376).
[33] "Der Begriff der **Energie** [...] beschreibt die Fähigkeit, Arbeit zu verrichten" (p. 129).
[34] "Energie kann in den verschiedensten Formen auftreten. Die *kinetische Energie* ist mit der Bewegung eines Körpers verbunden. Die *potentielle Energie* ist gespeichert Energie, die mit der räumlichen Anordnung der Bestandteile eines Systems zueinander zusammenhängt, beispielsweise mit dem Abstand zwischen einem Körper und der Erde. Die *Wärmeenergie* hängt mit der molekularen Bewegung innerhalb eines Systems zusammen und ist eng mit der Temperatur des Systems verknüpft" (p. 129).
[35] "Wärme ist die Energie, die von einem Körper auf einen anderen aufgrund einer Temperaturdifferenz übergeht" (p. 539).
[36] "Hier stellt sich die Frage "Was ist eigentlich Energie?" Doch wir können sie, ähnlich der nach dem Wesen der Kraft, nicht beantworten" (p. 109).
[37] "*Energie* kann weder erzeugt noch vernichtet werden, sondern *bleibt in jedem abgeschlossenen System konstant*. (Erfahrungstatsache!)" (p. 109).
[38] "Energie kommt in verschiedenen Formen vor, wie z.B. als Gravitationsenergie, kinetische Energie, Wärmeenergie, elastische, elektrische, magnetische, chemische Energie, Kernenergie und Strahlungsenergie. Bei den in der Natur ablaufenden Prozessen wird immer eine Energieform in eine andere umgewandelt, wobei jedoch die Summe der Einzelenergien konstant bleibt" (p. 109).

[39] "Thermodynamics can be defined as the science of *energy*" (p. 2).

[40] "Although everybody has a feeling of what energy is, it is difficult to give a precise definition for it. Energy can be viewed as the ability to cause changes" (p. 2).

[41] "**Heat** is defined as *the form of energy that is transferred between two systems (or a system and its surroundings) by virtue of a temperature difference*“ (p. 124).

[42] "*work is the energy transfer associated with a force acting through a distance*“ (p. 126-7).

[43] “Both are associated with a *process*, not a state” (p. 127).

[44] "Systems possess energy, but not heat or work” (p. 127).

[45] “In der Tat ist der Begriff der *Energie* so weit gefasst, dass eine einfache Definition nur sehr schwer zu geben ist. Zunächst einmal ist Energie eine skalare Größe, die mit dem Zustand eines oder mehrerer Objekte in Zusammenhang steht. Diese Definition ist jedoch zu vage, als dass sie eine echte Hilfe sein könnte“ (p. 152).

[46] “”Arbeit" ist also Energie, die übertragen wird [...] Der Begriff "übertragen“ kann auch irreführend sein. Er bedeutet nicht, dass etwas Materielles in das Objekt hinein- oder aus dem Objekt herausfließt; diese Energieübertragung darf man sich also nicht wie fließendes Wasser vorstellen. Sie entspricht vielmehr dem elektronischen Geldtransfer zwischen zwei Bankkonten: Die Zahl in dem einen Konto steigt an, während die Zahl in dem anderen Konto kleiner wird, obwohl zwischen den beiden Konten kein materieller Gegenstand ausgetauscht wird“ (p. 154).

[47] "This remarkable statement is the general form of the **law of conservation of energy**. In a given process, the kinetic energy, potential energy, and internal energy of a system may all change. But the *sum* of those changes is always zero” (p. 264).

[48] "When we expand our definition of energy to include internal energy, Eq. [...] says that *energy is never created or destroyed*; *it only changes form*” (p. 264).

References

1.2 Energy-History

History

In the history of science, we learn that energy was discovered in the 1840s: Mayer, Joule, Colding and Helmholtz are generally considered the discoverers. None of them were physicists at that time. Textbooks on physics usually refer to Mayer and Joule as discoverers of energy. Even though physicists and historians of science agree that Mayer and Joule discovered energy, their theories do not comply with each other. None of the discoverers speak of conservation of transformation of energy but of force.
The term energy was introduced by William Thomson in 1851. In 1852, a distinction in the stores of energy was introduced, which led to the division into kinetic and potential energy in the 1860s. Towards the end of the nineteenth century, the concept underwent a change in meaning: energy was understood as a substance by some physicists. An outline of this development will be given in order to clarify some difficulties with the concept nowadays.

Robert Mayer

In 1842, Mayer published an article in Liebig's *Annals of Chemistry and Pharmacy* with the title "Observations on the Forces of Inorganic Nature". He put forwards two questions: what forces are and how they are related to each other. The answer to the first is 'forces are causes'. This statement, which is not justified, is used to apply to forces the classical saying '*causa aequat effectum*', **cause=effect** (1842, p. 233). If the cause ***c*** has ***e*** as an effect and this has ***f***, then ***c=e=f***. On account of these equations, Mayer justifies two properties of force.
As ***c=e=f...,*** the quantity holds constant. Mayer says force is indestructible. As ***c=e***, Mayer says that ***c*** is transformed into ***e*** because when there is ***c*** at the beginning, ***e*** does not exist and when there is ***e*** no part of ***c*** exists. Forces are, therefore, quantitatively indestructible and qualitatively transformable, as he says (1842, p. 234). Let us move on to phenomena.
Firstly, Mayer deals with the falling of bodies. Contrary to the science of that time, he defends that weight is not enough for falling, for a body cannot fall without height (1842, p. 236). He defends instead that the weight and height of a body form together the cause of falling. The effect of this cause is motion. He takes for the cause the product of masse and height. Thanks to Galileo's law of falling, used by Leibniz's discovery of the conservation of the living force, the height is equal to the square of the velocity (1842, p. 236). Taking the product of mass and square of the velocity as the effect of the cause referred to, the

statement *cause =effect* leads to $mh = mvv$.

Heat-Motion

Concerning phenomena involving heat, Mayer firstly raises the question if a causal relationship between heat and motion can be established. It can be established if there are phenomena in which heat results from motion or heat produces motion. He set up an experiment to prove that motion causes heat: he agitated water in a recipient vehemently and the temperature of the water rose 12 or 13 degrees[1]. The steam-engine was given as an example of the inverse relationship: heat produces motion (1842, p. 239). Once admitted that there exists a causal relationship between heat and motion, an equation of the form

cause (motion) = effect (heat)

or

cause (heat) = effect (motion)

can be written. He used the second form.

The specific heat of atmospheric air at constant pressure is greater than the specific heat at constant volume. In the first case there is some motion and in the second there is none. Mayer considers the difference between the two quantities equal to the force performed in the variation of volume against atmospheric pressure. For calculation, he used the results of measurements known at that time.

Mayer calculated the difference between the specific heats at constant pressure and constant volume: Cp-Cv=0.000103 units of heat. This value represents the cause. Let us move on to the effect.

If one cubic centimetre of atmospheric air at 0° C is heated until 1° C, it raises a column of air of 1/274 cm. The weight of this column is 1033 g. The effect is therefore equal to 1033x1/274. As a weight of 1033 grams is lifted 1/274 cm by the expenditure of 0.000103 units of heat, one unit of heat is equivalent to 1 gram raised to 366 metres[2].

Electricity-Motion

The connection between electricity and motion is exemplified by an electrophorus. It constitues a base, a conductor and an isolating handle.

Replica of Lichtenberg's electrophorus, University of Oldenburg

If there has been friction on the base, an electrophorus produces an electric effect. Raising the upper part, a second effect can be obtained. Leaving the upper part to descend to the original position, an electric effect can be obtained again and raising the upper part, a second effect can be obtained once more. Mayer concludes that for each time a mechanical effect is made and an electrical effect is earned, the mechanical effect is transformed into electricity. The argument for this interpretation is based on another classical statement 'nothing comes from nothing'. He argues as follows. On one hand, the electricity from the lower part of the electrophorus is kept constant; on the other hand, a mechanical effect is produced. Then, either it is admitted that the mechanical effect does not have any consequence and that the electrical effects come from nowhere or it is concluded that the mechanical effect is transformed into electricity (1845, p. 24).
In conclusion, there are two steps in Mayer's dealing with the phenomena. Firstly, he verifies if a cause-effect relationship can be applied to the phenomena considered. Secondly, he writes an equation, whose sides are constituted by the quantities which characterise the cause and effect involved.

In 1848 Mayer published a book in his home town, Heilbronn, "Contribution to the celestial dynamic in a popular exposition". The origin of solar heat constitutes the central theme. This is developed in the following way: firstly, the possible sources of heat are considered; secondly, an estimation of the quantity of heat coming from the Sun is made; based on these two steps, a solution is proposed.
If there were a solar emission without renovation, there would be a cooling of the star of 9000° C in 5000 years, says Mayer (1848, p. 7-8). As this is not the case, there has to be renovation of heat. This renovation could happen whether by chemical or by mechanical processes. If it were chemical and imagining the Sun as a heap of coal, Mayer continues, the star could not have emitted heat for

more than 46 centuries (1848, p. 8). Therefore, having eliminated the chemical process hypothesis, the author moves on to considering the mechanical way: by friction, living force and impact.
Heat cannot be the outcome of friction, for Saturn spins faster than the Sun and does not give out such heat. Neither can it come from the living force of the Sun, due to its rotation, Mayer continues, for in such a case, it would not cover more than 183 years of heat output (1848, p. 10). Finally, Mayer's thesis is presented: the falling of bodies attracted by the sun is the origin of the sun's heat (1848, p. 12).
The argumentation is based on results of observations available and in calculations in which the mechanical equivalent of heat plays an important role. The first step of argumentation reminds the reader of the enormous and uncountable number of celestial bodies: planets, moons, comets and asteroids (1848, p. 14). From another piece of data from astronomy – there are bodies of which the diameter of the elliptical orbit diminishes - Mayer infers that the smaller the bodies, the faster their orbits diminish. Mayer finally concludes that heavy substances fall onto the solar surface (1848, p. 16).
The effect of the impacts on the solar surface would depend on the mass of the bodies but also on their speed. Mayer calculated the minimum and maximum speed that a body could hit the solar surface. Thanks to the mechanical equivalent of heat, he establishes a proportion between the velocity and a unit of heat. Thus, he concludes that an asteroid falling onto the Sun gives out between 4 600 and 9 200 more units of heat than a similar mass of coal (1848, p. 20).
He calculates then the quantity of matter which falls onto the sun: between 94000 and 188000 billion kilos per minute. This would represent an increase of 15 to 30 grams per square meter (1848, p. 31-32). Mayer reminds the reader that some light rain lasting an hour amounts to 17 grams per square meter (1848, p. 32).

In 1851 Mayer published in Heilbronn a book entitled "Observations on the Mechanical Equivalent of Heat". The work is divided in five parts. The first is concerned with the topic of the importance of numerical relationships in the research on nature. The golden rule of research, according to Mayer consists of acquiring knowledge of the phenomena before trying to explain them through ultimate causes (1851, p. 5-6). To know them, they should be researched until a numerical relationship is found. In this relationship resides the fundamental aspect of research, according to the author (1851, p. 7). The free fall is the first argument for this: the task of science consists of the establishment of a numerical relationship between height, duration of fall and velocity of falling; the experimental research leads to the result. The knowledge of the processes of combustion is another example: this was reached through the weighing of the products before and after the process. Another argument: the law of the development of heat in chemical reactions was reached by the establishment of a numerical relationship (see 1851, p. 7-12). Finally, the issue concerning heat developed by friction, inelastic impact or by pressure of gases was dealt with. Here too the task was not fulfilled with heat-substance, heat-ether and atoms of heat (1851, p. 12-13). It was the measurements of the work spent and the heat developed which allowed establishing a relationship between both. The number which establishes the relationship between the quantities of one and the other is the mechanical equivalent of heat (1851, p. 13-14).

To put forward the meaning of 'equivalent' the author uses an analogy from which we take the following sense: heat and motion are expressed in different units; the equivalent indicates how many units of one are necessary to obtain a certain number of units of the other (1851, p. 23).

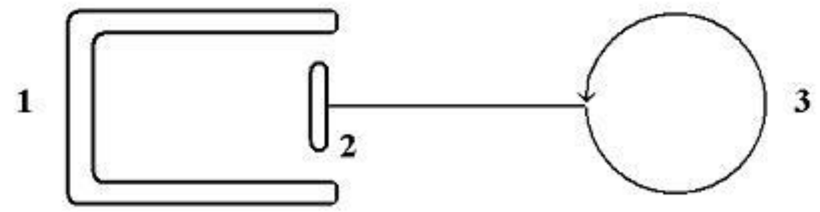
In the third part of the book, the use of the term force is debated. He pointed out that we do not know the real causes of phenomena but that what we have are the results of experiments. Referring to the types of forces, motion and force of falling, Mayer makes explicit the meaning of transformation of one force into the other: the concept does not mean more than the constancy of the relationship between both force of falling and motion (1851, p. 41-42). Concerning motion and heat, Mayer reinforces the statement of 1842 that heat and motion are different forms of force but this does not include that heat is motion. The connection between both motion and heat, the author stresses, is one of quantity and not of quality (1851, p. 43). What heat is can be seen as an open question, of which the solution would entail solving the question of ether and on knowing the essence of matter, namely if atoms exist, says Mayer (1851, p. 44). For this, he concludes that we know nothing about the essence of heat.

The fifth and last part of the work begins with an explanation of the issue and why the relationship between heat and motion was not discovered earlier. The reason of this delay lies in the terminology of physics. The designation of weight by force and of heat as a substance contributed to make the establishment of a relationship between heat and motion difficult (1851, p. 51). Finally, he shows the usefulness of the mechanical equivalent of heat in the resolution of problems which, without it, would remain unsolved – two examples of celestial physics are given (1851, p. 54-55).

James Joule

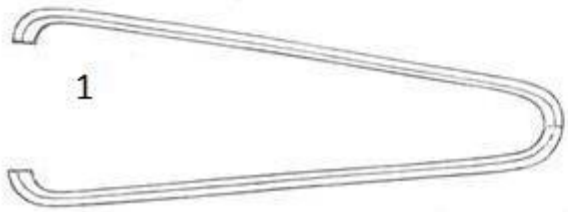
Joule published an article in the *Philosophical Magazine* of 1843, with the title "On the Calorific Effects of Magneto-Electricity, and on the Mechanical Value of Heat". 'Magneto-electricity' was the name given to the induced electric current, discovered by Faraday in 1831 (1884, p. 139). Joule constructed a magneto-electric machine.

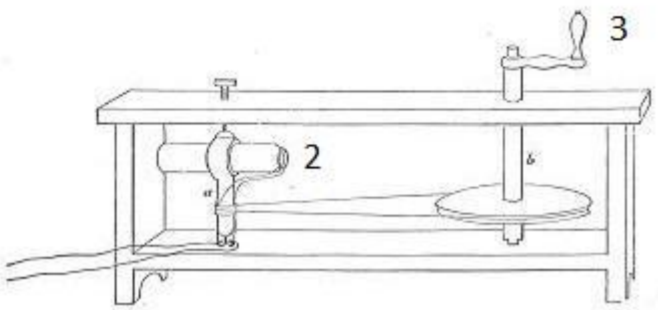
To have a clear idea of the machine, the following schema is useful (Coelho 2009, p. 967).



1 represents a magnet or an electro-magnet; 2-; an electro-magnet and 3- a crank which brings element 2 into rotation.

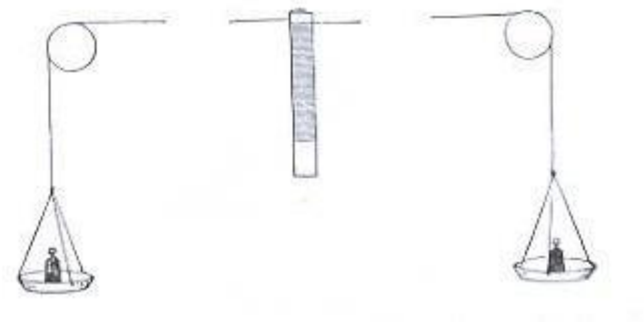
The following pictures, from Joule's paper, show element 1 and 2, 3.





If axle *b* is cranked, it brings axle *a* into rotation. Perpendicular to this axle, there is a hollow box, in which an electro-magnet is introduced. This element is immersed in a tube with water and thermally isolated. The temperature of the water is measured before the rotation and after it and the heat evolved is determined. From all the experiments set up with this experimental configuration, Joule concluded that motion creates heat.

He sets up some of the experiments again but now to find out a numerical relation between the mechanical power used in the motion of the machine and the heat evolved through the electric current. For this, he replaced the crank with the system represented in the following image (1884, p. 150).



To set the magneto-electric machine in motion with a velocity of 600 rpm, a weight of 5 lb 3 oz in each scale is necessary. In order to rotate with the same velocity but as a mere mechanical machine, therefore, without electricity production, 2 lb 13 oz are necessary. Joule took the difference of the two weights (4.75 lb) as the weight which contributes to the heat produced. As each scale covers the distance of 517 feet, the mechanical power is equal to
4.75 lb · 517 ft.

The heat evolved is equivalent to the heat necessary for heating a pound of water by 2.74 degrees Fahrenheit. If 2455.75 ft·lb corresponds to 2.74 units of heat, a one unit corresponds to x, where $x = 4.75 \times 517/2.74$. This is the mechanical equivalent of heat. Joule writes: "1° of heat per lb. of water is therefore equivalent to a mechanical force capable of raising a weight of 896 lb. to the perpendicular height of one foot" (1884, p. 151).
Thanks to that experimental configuration and others (Coelho 2009, p. 968-969), Joule obtained a total of thirteen results and proposed their average as the mechanical equivalent of heat, 838 ft·lb (1884, p. 156).

In 1845, Joule presented a new process to determine the mechanical equivalent of heat: the paddle-wheel experiment. The apparatus consists of a brass paddle-wheel working horizontally in a can of peculiar construction and filled with water.

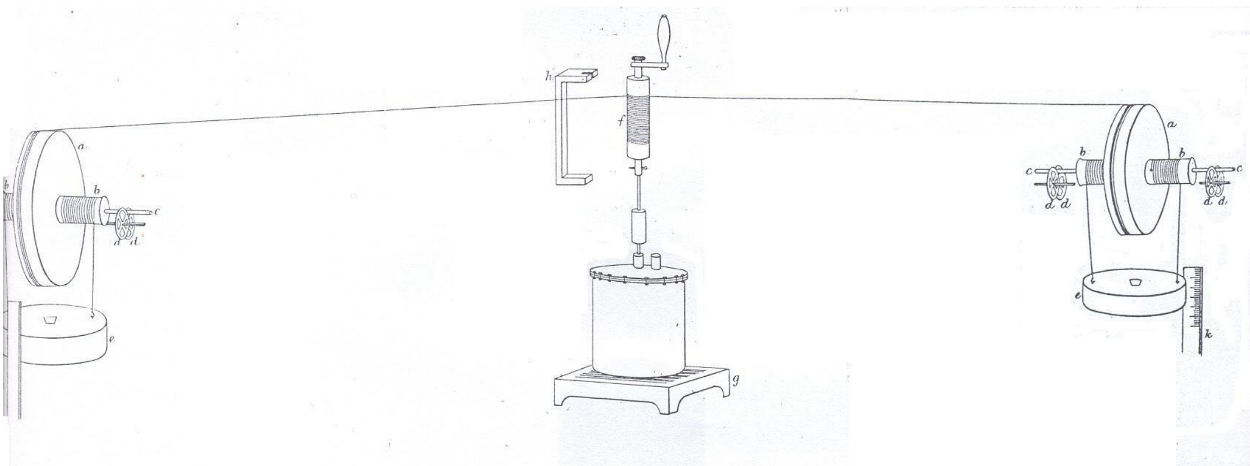
Replica of Joule's experiment, University of Oldenburg.

The paddle-wheel moves by means of weights thrown over two pulleys working in opposite directions.
Joule gave the following description of the experiments:
"The paddle moved with great resistance in the can of water, so that the weights (each of four pounds) descended at the slow rate of about one foot per second. The height of the pulleys from the ground was twelve yards, and consequently, when the weights had descended through that distance, they had to be wound up again in order to renew the motion of the paddle. After this operation had been repeated sixteen times, the increase of the temperature of the water was ascertained by means of a very sensible and accurate thermometer" (1884, p. 203).

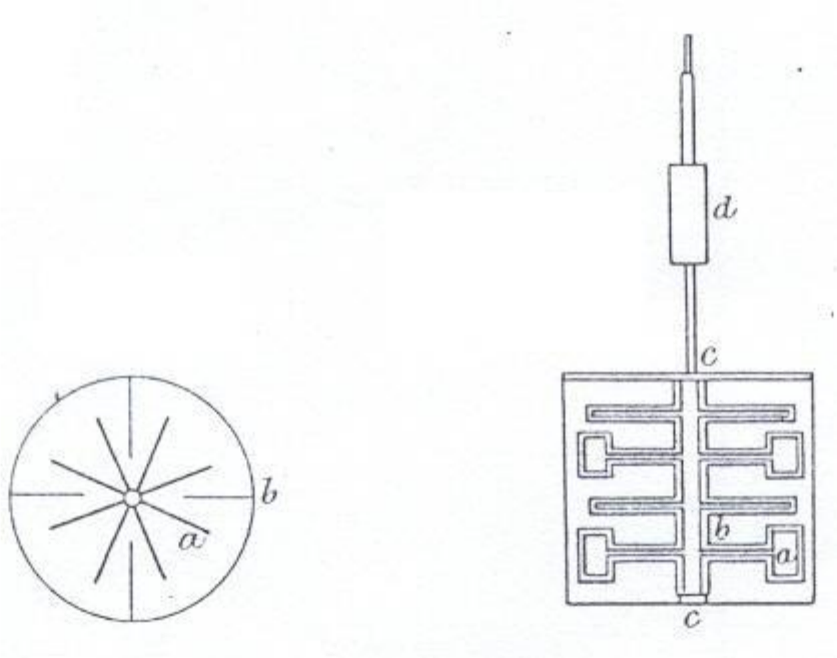
Replica: Detail of the inside of the can

The result was the following: "for each degree of heat evolved by the friction of water a mechanical power equal to that which can raise a weight of 890 lb. to the height of one foot had been expended" (1884, p. 203).

In 1850, an article on the same subject is published in *Philosophical Transactions*. In this, a schema of the paddle-wheel and of the construction of the can for the fluid is presented.



Joule's schema of the mechanism



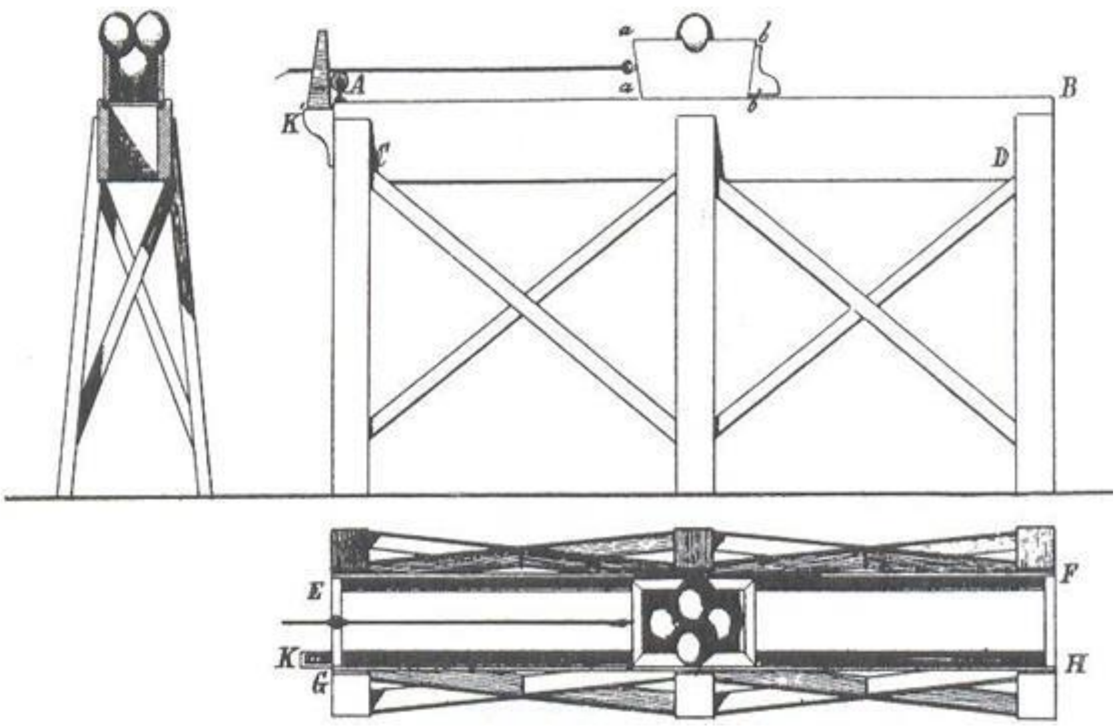
Joule's schema: detail of the inside of the can

Joule ended the paper with the two following propositions: "1st. *That the quantity of heat produced by the friction of bodies, whether solid or liquid, is always proportional to the quantity of force expended.* And, 2nd. *That the quantity of heat capable of increasing the temperature of a pound of water (weighed in vacuo, and taken at between 55° and 60°) by 1° Fahr. requires for its evolution the expenditure of a mechanical force represented by the fall of 772 lb. through the space of one foot*" (1884, p. 328).
A third proposition of the conclusion was suppressed in accordance with the wish of the Committee who reviewed the paper (see Philosophy).

Colding

'Theses Concerning Force' is the title of the talk, which would be later published in 1856, given by Colding to the Society of Sciences in 1843. If a force acts upon a particle and gives origin to movement, it is then propagated to the surrounding environment until it becomes impossible to detect. However, this is not a reason, argues Colding, to state that something has been lost without causing any effect. He thinks instead that it is of the very nature of things that action of forces which we perceive as vanished happens in a different fashion. He proposes as a general law of nature that when a force is apparently gone it goes through a transformation becoming then effective in other ways (1972, p. 1). The law is put forward as follows.
Assume that a given moving force, $\boldsymbol{q}$, has a totally lost effect. Assume there has been a transformation and that the new action, in which the force is manifested, is equal to $\boldsymbol{q}$. Now the moving force, adds Colding, normally gives an amount of

heat. Therefore, to the given heat the same value ***q*** will be ascribed (1972, p. 1-2). The experiments of other authors, referred to by Colding, as well as his own experimental work are concerned with the relationship between force and heat. The types of work which conform to his thesis would be the studies upon compression of gases (Dulong), of liquids (Colladon and Sturm, Oersted) of solids (Berthollet); the rupture of iron sticks (Lagerhjelm); and the heat obtained by friction (Rumford, Haldat, Morosi). The available results obtained have been, however, insufficient to prove the thesis, Colding states, and for that reason he works on laboratory research. The reader is nevertheless informed ahead of time that his instrument is not sufficiently finalized (1972, p. 3). Colding does experiments on the friction of solids. He uses a similar mechanism that had been used by Coulomb in his study of machines. The next picture gives us an image of the instrument from the front, from the side and from the top.



From Colding's paper

The instrument is made up of two parallel bars made of tin of a little more than two meters long: on this, a small sledge slides. The sled, represented in the middle of the line AB, is pulled by hand by a string which is connected to it on the edge. The sledge carries balls, as it is suggested by the picture, and the quantity of these may make its weight vary. The distance covered by the sliding is the same in all experiments and the course is run twice. The author says that the speed of the course is more or less the same in all experiments (1972, p. 4). The dilation of the threads on the base of the sledge gives an indication of the heat produced. For this, Colding has a small instrument, which reacts to the difference of length and accounts for the variation of heat. This is connected to the sledge and another one to one of the bars.
Colding did 10 series of experiments. The difference between them lies in the materials constituting the sledge and the threads: tin, zinc, lead, linden wood, linden wood wrapped with flannel and iron (1972, p. 6-11). In each of the series, the moving force to pull the sledge is measured thanks to a dynamometer and the produced heat of by the devices referred to. Let us consider in more detail the first three series, which were all done with threads of tin and in which only the weight of the sledge changes: 88.75, 53.5 and 31 pounds each different time. In the first series, there were 32 measurements made, in regard to which an average value of force and of the developed heat was calculated. The final result was of 30.3 pounds and 2 degrees. In the second series, there were 50 measurements and the obtained average values were of 19.7 pounds for force and 1.32 degrees of heat. In the third series, there were 56 measurements leading to 11 pounds and 0.72 degrees of heat. “11” and “0.72” will be taken as the unit in the following sense: it is calculated the ratio among all the values and these. Thus, the proportion between 30.3 lb. of the first series and 11 lb. of the third is of 2.75; the proportion between 19.7 lb. of the second and the eleven from the first is 1.79.
Proceeding in a similar way with the results of heat, Colding obtains the ratios between 2 degrees of the first series and 0.72 degrees of the third, which gives the result of 2.77; and between 1.32 of the second and 0.72, which gives 1.83. Collecting all the data, we obtain:

Moving force: ....2.75 ....1.79 1
Heat developed: ....2.77 ....1.83 1

The results of the following series are treated in the same fashion and placed in a crescent order of values we have:

Moving force	1	1.24	1.68	1.74	1.77	1.79	2.75
Developed heat	1	1.20	1.66	1.80	1.76	1.83	2.77

Beyond these results, there are three more that the author does not consider capable of being valuable. In compliance with the ratio between force and heat, Colding concludes that the amount of heat developed in all cases is proportional to the loss of moving force (1972, p. 12).
In consequence of these experiments, as well as others regarding the development of heat through force, Colding considers his thesis sufficiently proved. He goes as far as to consider that it is valid for all types of forces (1972, p. 13). One last argument is given of the "*perpetuum mobile*": the thesis will be necessary for the proof of its impossibility (1972, p. 14).

Helmholtz

In 1847, Helmholtz gave a talk to the Physical Society of Berlin "On the Conservation of Force", which was published at his own cost in the same year. The piece can be divided into three parts: firstly, the theoretical basis of Helmholtz's dealing with phenomena is given; in the second, this dealing leads to the conservation of force, which is developed in the various domains of physics – mechanics, heat theory, electricity and magnetism; the third, shorter than the others, is concerned with living beings.
The main thesis is that there are two ultimate or fundamental forces in nature, living force and force of tension, whose addition is constant. It is this constancy that is referred to by 'conservation of force' in the title of the work. Essentially, the argumentation consists of showing that phenomena are consistent with the assumption of the two forces. We shall begin with the first part, where the ground for considering the phenomenon in light of the two forces is put forward.
The task of the theoretical part of science should consist of the search for the ultimate causes of phenomena according to the law of causality. This means that if the causes by which phenomena are explained are mutable, explains Helmholtz, one should continue the search for the ultimate causes until they are found. This demand is justified by the principle of sufficient reason (1882, p. 13). Given this, Helmholtz begins characterizing the ultimate causes. In order to do this, he begins by considering how objects are viewed by the knowing subject. In science, objects of the external world are considered according to a double abstraction: matter and force. Matter as such does not have an effect on our sensory organs; the effects that external objects have on us are forces. Consequently, it would be erroneous, Helmholtz continues to explain, to consider matter real and force not. Both matter and force are abstractions and are inseparable, argues the author, for it is based on force that we infer the existence of matter (1882, p. 14).
From here, the first statement in the consideration of the search for

determining the task of theoretical science is taken: just as the existence of matter is inferred from force, the return to the ultimate causes means a return to immutable forces. Well, according to the meaning of force, immutable forces will be those that reach the sensory organs in an invariable fashion through time. According to the concept of force as well, what arrives to us in a constant fashion reflects what is permanent in matter.

This allows us to understand Helmholtz, who takes 'immutable forces' as a synonym of 'indestructible qualities of matter'. These qualities would have been determined by chemistry and would be the elements (1882, p. 15). If we think of the world, writes Helmholtz, as constituted by elements of indestructible qualities, any change in it would be special and would become motion. In other words, as we are facing elements, which in themselves do not change, the only changes that can occur are changes in their relative positions, that is, motions. From here, another determination of the task of theoretical science is taken: as any change can only come from movements, the phenomena must be traced back to movements of matter, from which the moving forces can only depend on spatial relationships. The next step consists of the determination of this dependency. Motion is defined as the change of spatial relationships. These relationships can only take place in a limited space, writes Helmholtz, where at least two bodies exist. Due to this consideration and to the definition of motion, it is understood that the forces of motion are seen as a tendency of two masses of changing their relative positions (1882, p. 15). As abstracting from rotation, as is the case, the approximation or distancing of the two masses is what is left to us, the last characterization of forces is: they are either attractive or repulsive.

Helmholtz arrives in this way to the delimitation of the final task of theoretical science: the tracing back of phenomena to immutable forces of attraction or repulsion in the direction of the line that unites the two masses and of which intensity depends on the distance between those bodies (1882, p. 16). Once the task is established, the author moves on to practical concerns and considers the treatment of phenomena.

Helmholtz begins by introducing the assumption which, according to him, Carnot and Clapeyron had used successfully, which states that it is impossible to have a durable force out of nothing. The proposition is transformed into a principle: it is assumed that a moving force and its resulting movement are equivalent (1882, p. 17). The mathematical expression for this is based on the principle of conservation of living forces. The equation states:

$$\frac{1}{2}mv^2 = mgh$$

where *m* represents the mass of the body, *v* the velocity, *g* the gravitational acceleration and *h* the height. 1/2mvv is seen as a measure of the work and is called living force, instead of mvv, as it had been called so far (1882, p. 18). (In this section we will keep the helmholtzian sense of living force.) The next step consists of the generalization of the law for every central force. The difference between the living force at the end and at the beginning of a path is explained by the acting forces. Writing the intensity of the acting force as φ, the mathematical expression assumes the form:

$$\frac{1}{2}mv^2_{(final)} - \frac{1}{2}mv^2_{(initial)} = -\int_r^R \varphi dr,$$

where the 'minus' of the integral is conventional: the intensity of the force is taken as positive in the attraction and negative in the repulsion (1882, p. 21-2).

This equation is then read in the following way; the increase of the living force of a moving point under the influence of a central force is equal to the sum of the forces of tension. Consequently, the principle of the conservation of force states: the sum of the living forces and of the forces of tension is constant (1882, p. 24-25).

The principle of conservation of force is then defended for the other domains of physics. When Helmholtz treats phenomena which involve heat, he starts by approaching the cases in which it was considered that a loss of force had taken place: the inelastic impact and friction. The argumentation is as follows. In an impacted body, there is a change of form and a change in density. These changes cause an increase of the force of tension. Moreover, when the impact is repeated as for example, when a metal is hit with a hammer, heat is detected. On the one hand, part of the effect passes on into the air as with sound. Concerning friction, in addition to the change of form and of the movement of particles away from the surface, there are changes in the thermal and electric conditions (1882, p. 32).

In sum, concerning the absolute loss of force, the author, on the contrary, defends that there are some effects. Nevertheless, for the acceptance of the conservation thesis, it is not sufficient that effects take place but it is also necessary that the sum of the forces be constant.

The author then begins to deal with this demand considering the two following questions: 1) if, in the loss of mechanical force, a certain amount of heat appears; 2) in what sense can a quantity of heat correspond to a mechanical force (1882, p. 32-33).

The answer to the first question is positive. The argument given comes from Joule's experiments of 1843 and 1845. Joule's mechanical equivalent of heat is put into question but the existence of equivalence is accepted (1882, p. 33). The second question can be understood as the questioning of how heat can generate work.

Helmholtz states that if we assume that heat is matter, the production of work will have to have origin in the expansion of the substance. This thesis is not directly denied. The author moves on instead to show that the concept of heat as a substance is not adequate to explain the thermal phenomena in general. The explanation of heat by friction is the first difficulty pointed out by Helmholtz. The theory would have to assume either that heat comes from the exterior (Henry) or that it is caused by compressions on the surface (Berthollet) (1882, p. 33-34). The first explanation would be inconvenient due to lack of experimental evidence; the second, Helmholtz continues arguing, is in opposition to experience, as Davy's work on the melting of ice by friction showed. Another argument against the theory of heat as a substance points out that the quantity of heat can be increased absolutely through motion (1882, p. 34).

Here, Joule's experiment with the magneto-electric machine is called upon: while there is movement, heat appears. Well, the author continues, heat can be obtained indefinitely, and cannot come from the pieces of the machine. From the facts, where heat is obtained thanks to mechanical force, the author concludes that such phenomena cannot be explained by heat as a substance. He adds they are to be explained by the concept of heat as motion (1882, p. 35). Within his theoretical framework, the amount of heat is to be understood as the sum of the living forces of heat and the forces of tension in the atoms. The heat connected with the living forces would correspond to the so called free heat; and the heat connected with the forces of tension would be the so called latent heat. From the motion of atoms, an idea is given but it is presented as a mere hypothesis. For the author it is enough that the phenomena of heat can be thought of in terms of motion (1882, p. 36). Let us move on to the equivalent of force in electric phenomena.

Take two unit electric charges of opposite signs. The force between them is proportional to the charges and inversely proportional to the square of distance. The variation in living force when they are brought within a certain distance of each other is given by the force of tension taking place (1882, p. 41). Let us take just one of the electric current cases, the thermo-electricity. Helmholtz refers to a phenomenon discovered by Peltier: if a current passes through two metals, which are connected to each other at one extremity, at the joining place there will be warmth or coolness depending on the direction of the current. The law of conservation leads to an equation in which the left-hand side represents the heat from the source, i.e. from the battery, and the right-hand side the heat of the remaining circuit. The right-hand side consists of the heat which is developed in the circuit, according to the law of Joule or Lenz, plus the heat that disappears or appears at the joining point of the metals. The equation has the following form (1882, p. 57).

$$AJ = J^2W + q_1 - q_2$$

where A represents the electromotive force, J the intensity of the current, W the resistance, q1 and q2 the developed and disappeared heat. From the expression Helmholtz draws consequences which are not compared with experimental results, because such experiments had not yet been made. Let us move on to magnetism and electromagnetism.
Take two magnets which repel each other. The force between them is inversely proportional to the square of distance, more precisely expressed by

$$\varphi = -\frac{(m_1 m_2)}{r^2}$$

The integral of force over distance gives the respective living force (1882, p. 59).

Take now the movement of a magnet under the influence of a current. In this case, we have to consider the battery, which is the origin of the current of the circuit where the current goes through, and the magnet. Helmholtz places on one side of the equation what concerns the battery and on the other side, what remains. The heat developed in the battery and circuit are known, namely A*Jdt* and *JJWdt*. These are expressions of heat and not of mechanical magnitudes. Helmholtz transforms them by multiplying them by the mechanical equivalent of heat. Using '***a***' to represent the mechanical equivalent of heat, we will have *aAJdt* for the battery and *aJJWdt* for the current in the circuit. The first is considered force of tension and the other living force. The term used for the magnet is given by J *d*V/*dt dt*, where V is the potential in relationship to the unit of the current. Placing the terms altogether we obtain

$$aAJdt = aJ^2Wdt + J\frac{dV}{dt}dt.$$

Helmholtz takes out the value of J. Instead of J = A/W, as it was known, he gets

$$J = \frac{A - \frac{1}{a}\frac{dV}{dt}}{W}.$$

The new term *d*V/*a dt* is interpreted as a new moving force, the force of inductive current (1882, p. 62).

The final part, and the shortest in the work, refers to living beings. The plants would have a great quantity of forces of chemical tensions and would absorb a single form of living force while growing, namely the radiation of solar light.

There were no indications that allowed expressing the equivalent of force. In relation to animals, there were more indications. They spend a certain amount of quantity of forces of chemical tensions and produce motion and heat. Helmholtz concludes, however, that the indications are not sufficiently precise for the discussion of conservation of force (1882, p. 66).

The term energy

Energy, etymologically, means activity. The word was commonly used in the eighteenth century. In 1851, William Thomson uses the word to refer to the mechanical activity of a body, i.e., what a body can do mechanically. This is the meaning of his concept of "mechanical energy of a body". The "total mechanical energy of a body" is the mechanical activity of a body, which depends on the total heat in it, i.e, the mechanical value of all effects a body would produce in heat emitted and in resistances overcome, if it were completely cooled[3]. As this is impossible to determine[4], Thomson defines the mechanical energy of a body in a given state[5]. By energy of a body is then understood the mechanical value of the effects the body would produce in passing from that state to a standard one or from this to the given state (see Coelho 2009, p. 973-977). This concept of energy and the fact that phenomena are irreversible led him to the thesis that there is a universal tendency to the dissipation of mechanical energy[6]. He argues: only the "Creative Power" can annihilate mechanical energy; as the phenomena are irreversible, there is an absolute waste of mechanical energy available for man; therefore a transformation must have taken place[7].

"To explain the nature of this transformation, he writes, it is convenient, in the first place, to divide *stores* of mechanical energy into two classes - *statical* and *dynamical*. A quantity of weights at a height, ready to descend and do work when wanted, an electrified body, a quantity of fuel, contain stores of mechanical energy of the statical kind. Masses of matter in motion, a volume of space through which undulations of light or radiant heat are passing, a body having thermal motions among its particles (that is, not infinitely cold), contain stores of mechanical energy of the dynamical kind" (1852, p. 139).

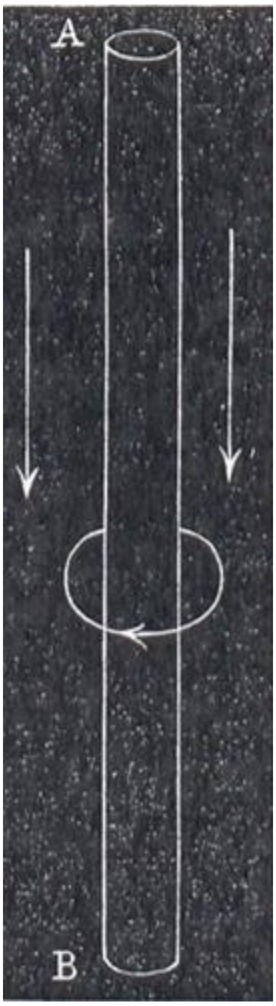
In the following year, Rankine proposed a new terminology for the systematization of energy: "All conceivable forms of energy may be distinguished into two kinds; actual or sensible, and potential or latent. *Actual energy* is a measurable, transferable, and transformable affection of a substance, the presence of which causes the substance to tend to change its state in one or more respects [...] by the occurrence of which changes, actual energy disappears, and is replaced by *Potential energy*" (1853, p. 106).

Thomson adopted Rankine's terminology as follows: "The energy of motion may be called either "dynamical energy" or "actual energy". The energy of a material system at rest, in virtue of which it can get into motion, is called "potential energy."" (1854, p. 34). The concept 'kinetic energy' appeared for the first time in a non-scientific review, *Good Words*, 1862: "[...] It had kinetic or (as it has sometimes been called) actual energy. We prefer the first term, which indicates motion as the form in which the energy is displayed" (p. 602). Some years later Thomson explained this change as a consequence of a terminological reform: "in advocating a restoration of the original and natural nomenclature, - "mechanics the science of machines," – "dynamics the science of force," I suggested (instead of statics and dynamics the two divisions of mechanics according to the then usual nomenclature) that statics and kinetics should be adopted to designate the two divisions of dynamics. At the same time I gave, instead of "dynamical energy," or "actual energy," the name "kinetic energy" which is now in general use to designate the energy of motion" (1884, Vol. II, p. 34).

In his *Theory of Heat* Maxwell defends that heat is a form of energy for the following reason: "heat may be generated by the application of work, and that for every unit of heat which is generated a certain quantity of mechanical energy disappears" (1873, p. 93). Energy of a body is defined as its capacity of doing work[8].

This definition was criticised by Oliver Lodge 1879, who instead proposed to understand energy as the work done on a body. He writes: "This definition of energy, as the effect produced in a body by an act of work, is not so simple as the usual one – "the power of doing work" but this latter definition seems a little unhappy" (1879, p. 279). This is explained by the following analogy: "energy is power of doing work in precisely the same sense as capital is the power of buying goods. [...] money is a power of buying goods. It does not, however, *necessarily* confer upon its owner any buying-power, because there may not be any accessible person to buy from; and if there be, he may have nothing to sell. Just so with energy: it usually [...] confers upon the body possessing it a certain power of doing work, which power it loses when it has transferred it" (1879, p. 279).
The conservation of energy is then expressed in the form: "energy is neither produced nor destroyed, but is simply transferred" (1879, p. 279).

The following picture is from Poynting's paper "On the Transfer of Energy in the Electromagnetic Field", 1884.



From Poynting's paper, 1884

If a current goes through a wire the vector product of the "electromotive intensity" and the "magnetic intensity" leads to the expression of the heat developed. He writes: "Let *r* be the radius of the wire, *i* the current along it, *α* the magnetic intensity at the surface, P the electromotive intensity at any point within the wire, and V the difference of potential between the two ends. Then the area of a length *l* of the wire is 2rl, and the energy entering from the outside per second is.

$$\frac{areaxE.M.I.xM.I.}{4\pi} = \frac{2\pi rl.P.\alpha}{4\pi} = \frac{2\pi r\alpha.Pl}{4\pi} = \frac{4\pi iV}{4\pi} = iV$$

for the line integral of the magnetic intensity 2rα round the wire is 4x current through it, and P*l*=V.
But by Ohm's law V=*i*R and *i*V=i^2R, or the heat developed according to Joule's law" (p. 350-1).
As heat is a form of energy and from a formal point of view the result of the calculation is a vector perpendicular to the other two, Poynting interpreted the phenomenon in the following way: "In this case very near the wire, and within it, the lines of magnetic force are circles round the axis of the wire. The lines of electric force are along the wire [...] energy is therefore flowing in perpendicularly through the surface, that is, along the radius towards the axis" (1884, p. 350).

Based on this paper, Lodge developed his concept of energy: "whenever energy is transferred from one place to another at a distance, it is not to be regarded as destroyed at one place and recreated at another, but it is to be regarded as transferred, just as so much matter would have to be transferred; and accordingly we may seek for it in the intervening space, and may study the paths

by which it travels" (1885, p. 482).
Energy became something substantial: “the common mode of treating a falling weight, saying that its energy gradually transforms itself from potential to kinetic but remains in the stone all the time, is, strictly speaking, nonsense. The fact is the stone never had any potential energy, no rigid body can have any; the gravitation medium had it however, and kept on transferring it to the stone all the time it was descending" (1885, p. 486).

In 1887, Planck was aware that had been understood as a substance (1921, p. 116). There was however a difficulty with this concept. As the energy of an isolated system remains constant, the same quantity of energy must be there. However, says Planck it is not possible to localise the energy in the system. This concept of energy was considered by him as a concept which one day should be overcome[9].

Also Hertz, 1894, was aware that energy had been understood as a substance. He criticised this concept of energy. The reason for this criticism is the following: it is not logically permissible to understand energy as a substance, since energy has properties which contradict the concept of substance. This means for instance the following: “The amount of a substance, writes Hertz, is necessarily a positive quantity; but we never hesitate in assuming the potential energy contained in a system to be negative”; “the amount of any substance contained in a physical system can only depend upon the state of the system itself; but the amount of potential energy contained in given matter depends upon the presence of distant masses which perhaps have never had any influence upon the system” (2003, p. 23).

Notes

[1]"Wasser erfährt, wie der Verfasser fand, durch starkes Schütteln eine Temperaturerhöhung. Das erwärmte Wasser (von 12° und 13°C.) [..]" (1842, p. 238).

[2]"Ein Kubikcentimeter atmosphärische Luft bei 0° und 0m,76 Barometer, wiegt 0,0013 Gramme; bei constantem Drucke um 1°C. erwärmt, dehnt sich die Luft um 1/274 ihres Volumens aus und hebt somit eine Quecksilbersäule von einem Quadratcentimeter Grundfläche und 76 Centimeter Höhe um 1/274 Centimeter.
Das Gewicht dieser Säule beträgt 1033 Gramme. Die specifische Wärme der atomsphärischen Luft ist bei constantem *Drucke*, die des Wassers =1 gesetzt, nach ***Delaroche*** und ***Bérard*** = 0,267; die Wärmemenge, die unser Kubikcentimeter Luft aufnimmt, um bei constantem *Drucke* von 0 auf 1° zu kommen, ist also der Wärme gleich, durch welche 0,0013 x 0,267 oder 0,000347

Gramme Wasser um 1° erhöht werden. Nach ***Dulong*** [...] verhält sich die Wärmemenge, welche die Luft bei constantem Volumen aufnimmt, zu der bei constantem Drucke, wie 1:1,421; hiernach gerechnet ist die Wärmemenge, die unseren Kubikcentimeter Luft bei constantem V*olumen* um 1° erhöht, =0,000347/1,421 = 0,000244 Grad. Es ist folglich die Differenz *y*=0,000347-0,000244=0,000103 Grad Wärme, durch deren Aufwand das Gewicht P=1033 Gramme auf *h*=1/274 Centimeter, gehoben wurde. Durch Reduktion dieser Zahlen findet man 1° Wärme=1Grm. auf 367m [...] Höhe" (1845, p. 14-5).

[3] “The total mechanical energy of a body might be defined as the mechanical value of all the effect it would produce, in heat emitted and in resistances overcome, if it were cooled to the utmost, and allowed to contract indefinitely or to expand indefinitely according as the forces between its particles are attractive or repulsive, when the thermal motions within it are all stopped” (1851, p. 475).

[4] “in our present state of ignorance regarding perfect cold, and the nature of molecular forces, we cannot determine this “total mechanical energy” for any portion of matter” (1851, p. 475).

[5]"the "mechanical energy of a body in a given state,” will denote the mechanical value of the effects the body would produce in passing from the state in which it is given, to the standard state, or the mechanical value of the whole agency that would be required to bring the body from the standard state to the state in which it is given” (1851, p. 475).

[6]"There is at present in the material world a universal tendency to the *dissipation* of mechanical energy" (1852, p. 141).

[7]"As it is most certain that Creative Power alone can either call into existence or annihilate mechanical energy, the "waste" referred to cannot be annihilation, but must be some transformation of energy" (p. 139).

[8] “the energy of a body may be defined as the capacity which it has of doing work” (p. 90).

[9]"Gewiß ist zuzugeben, daß diese (sozusagen materielle) Auffassung der Energie als eines Vorrats von Wirkungen, dessen Menge durch den augenblicklichen Zustand des materiellen Systems bestimmt ist, möglicherweise später einmal ihre Dienste getan haben und einer anderen, allgemeineren und höheren, Vorstellung Platz machen wird: gegenwärtig ist es jedenfalls Sache der physikalischen Forschung, diese Auffassung als die anschaulichste und fruchtbarste überall bis ins einzelne durchzubilden und ihre Konsequenzen an der Hand der Erfahrung zu prüfen" (p. 118).

1.3 Energy-Philosophy

Philosophy

Mayer

There are two steps in Mayer's dealing with the phenomena. Firstly, he verifies if a cause-effect relationship can be applied to the phenomena considered. Secondly, he writes an equation, whose left-hand side and the right one are constituted by the quantities which characterise the cause and effect. The 'cause' and the 'effect' in phenomena are called forces. This differs from the concept of force in the science of that time (Force). Mayer justifies his new concept based on his concept of cause. Moreover, he pointed out that, for instance, 'transformation of heat into mechanical effect' expresses a fact and does not explain the transformation. As we say, he exemplifies, that ice is *transformed* into water without knowledge of *how and why* it happens, it is also said that heat is transformed into mechanical effect. He stressed that the relationship between heat and motion is quantitative and not qualitative[1]. Furthermore, he criticised the question of what is going in such physical processes as useless and typical of poets and philosophers of nature[2]. In sum, Mayer did not discovery a kind of force which is transformable and cannot be destroyed. Instead, he uses what is observable or measurable. Hence, he does not need to discuss, for instance, the nature of heat, as was usual at that time.

Joule

The following "equation" subsumes the case considered:

(*Weight* (mag.-elec.machine) - *Weight* (mec.machine)) *x Height* = *Heat*.

We can also write it in the form

α units of mechanical power = β degrees of heat.

Thanks to this, the mechanical equivalent of heat is calculated. Let us move on to the interpretation.
Joule’s experimental work aims to find a solution for the question of “whether the heat observed was *generated*, or merely *transferred from the coils* in which the magneto-electricity was induced, the coils themselves becoming cold" (p. 123).
If it is generated, then it would be difficult to conceive of heat as a substance:”when we consider heat not as a *substance*, but as a *state of vibration*, there appears to be no reason why it should not be induced by an action of a simply mechanical character, such, for instance, as is presented in the revolution of a coil of wire before the poles of a permanent magnet" (p. 123). As the experiments show that heat can be generated, it could not be a substance. According to the initial dilemma, it is then a kind of motion. Once this is admitted, then it is understandable that the mechanical equivalent of heat is considered a convertor between those two kinds of motion: the mechanical one and the state of vibration which characterizes heat.

On the paddle-wheel experiment
The 1850 paper opens with quotations concerning the thesis ‘heat is motion’ and a reference to living force:
- Locke: ""Heat is a very brisk agitation of the insensible parts of the object, which produces in us that sensation from whence we denominate the object hot; so what in our sensation is *heat*, in the object is nothing but *motion*."
- Leibniz: "The *force* of a moving body is proportional to the square of its velocity, or to the height to which it would rise against gravity."
- Rumford: “[...] it was reserved for Count Rumford to make the first experiments decidedly in favour of that view [...] "It appears to me," he remarks, "extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner the heat was excited and communicated in these experiments, except it be motion" [...]" (p. 298-9).
- Davy: "By rubbing two pieces of ice against one another in the vacuum of an air-pump [...] This experiment was the more decisively in favour of the doctrine of the immateriality of heat, inasmuch as the capacity of ice for heat is much less than that of water. It was therefore with good reason that Davy drew the inference that "the immediate cause of the phenomena of heat is motion, and the laws of its communication of motion [...]" (p. 300).
The conclusion of the paper is presented in two propositions. A third one was suppressed by the committee who reviewed the paper. The published propositions express results of the experimental work. The third one concerns Joule’s explanation or interpretation of the phenomenon. Let us consider the two aspects involved: experiment and interpretation.

At the beginning of the experiment, the weights are at a certain height and the water is a certain temperature.
At the end, the weights are down and the temperature of the water has increased.
Thanks to these data the following relationship is established

α mechanical units= β units of heat

and the mechanical equivalent of heat calculated.
Is this a phenomenon of conversion from mechanical power into heat, as Joule interpreted it?
If there is a conversion, it must have taken place within the can. No observation is made to verify what happened within it during the conversion process, presuming this takes place. On the other hand, what really happened within the can during the falling of the weights is also not important for the final result. If the phenomenon does not consist of a process of conversion and heat is not a kind of motion, the established relationship between heat and mechanical power is not disturbed. The conversion is therefore an interpretation of the phenomenon.
If we now ask the question of what Joule discovered, it could be answered that he found *experimental methods for determining the mechanical equivalent of heat.*

Colding

The experiments of Colding can be represented by the following scheme

where the small segment represents the sled and the segment AB represents the bars in which it is moved. The distance covered is the same in all cases, and the speed of the path is approximately the same, the variety of experiments has to do with the materials used in the threads of the sledge and the weight used. The force used to pull the sledge is measured by a dynamometer and the expansion of the bars and threads are measured through a sensor which allows the reading of the heat developed. Colding establishes a proportion between the increase of force and the increase in the heat. The results of 7 of the ten series of experiments show that a greater force corresponds proportionally to a greater heat. This could be written:

Variation of force (Δ F) *is proportional to variation of heat* (Δ C) or

$$\frac{\Delta F}{\Delta C} \approx 1$$

On the interpretation of the results

The experiments are made to test the idea of the author that forces of nature are imperishable. Observation shows that forces disappear but Colding puts forward the thesis that they are not destroyed but transformed. The elements of this transformation are observable, such as motion and heat. If one considers the thesis of the imperishableness of natural forces and the result obtained by Colding, it is realized that the contribution of the result for the thesis lies in the following: the more force that is made, the more heat appears; the force added does not disappear but becomes heat. If Colding had used more force and the heat developed had remained the same, he would not have an argument to defend the imperishableness. The experimental work gives him a reason for the thesis, the result corroborates the imperishableness. The mathematical dealing with the elements of a transformation, proposed by Colding, is the following: if what is given at first is capable of being represented quantitatively by ***q***, the effect must be equal to ***q***. The equation proposed was, however, not applied. As the moving force disappears one would expect an equation of the type ***q***=β units of heat, taking ***q*** as the value of the force that disappears. What Colding presents is a proportion that says that if the force increases in 2.75 units then the heat increases in 2.77.

In conclusion, Colding obtained technical information that says that if the dynamometer indicates about 15 pounds more than in a previous experiment then one can expect one unit of heat more. This datum is connectable with the thesis of the imperishableness of the forces of nature in the sense described above. Imperishableness and transformation are elements of interpretation that are part of the original idea of the author.

In this knowledge, previous to the experimental work, the following can be observed: the imperishableness justifies the mathematical dealing initially proposed. Given that the forces do not die, their initial quantity ***q*** cannot diminish. The transformation has for function relating what is diverse in observation, such as motion and heat. What does not die and is transformed is in itself something extraordinary but the label 'forces of nature' allows accepting very special properties.

Helmholtz

Let us consider how Helmholtz dealt with the phenomena presented in the historical part. The falling of bodies is the paradigmatic phenomenon. The author took the weight in height as the cause of the falling motion. The cause is designated force of tension, the effect living force and Helmholtz arrives at the expression of

Δ forces of tension = Δ living forces.

This is the expression that constitutes the form of the remaining equations. Thus we get:
– for the paradigm itself

$$\frac{1}{2}mv^2 = mgh$$

– in static electricity the gain of living force is given by

$$-\int_R^r \varphi dr = \frac{e,e,,}{R} - \frac{e,e,,}{r}$$

– in heat phenomena, the motion is not visible yet the latent heat is seen as force of tension and the sensitive heat as living force;
–in thermo-electricity holds

$$AJ = J^2W + q_1 - q_2$$

where the left-hand side is connected with forces of tension and the second with living forces;

– in electromagnetism, force of tension = living force (heat) + living force of magnet,

$$aAJdt = aJ^2Wdt + J\frac{dV}{dt}dt\,.$$

In the case of falling, the force of tension and the living force are connected with observable data; in the case of heat that is no longer the case. In the cases where observation is not available, Helmholtz uses observable elements and interprets them. For instance, the battery, which feeds the circuit, is connected with the force of tension and the remaining circuit with the living force. The justification to apply the conceptual scheme based on falling to an unobserved field is given by the initial theoretical construction, namely, that those ultimate forces exist and that the task of physics is to trace the phenomenon back to them. As these forces are of mechanical nature, the mechanical equivalent of heat enables Helmholtz to pass on to mechanical units what was expressed as heat units.

On the discovery of energy

The expression

α mechanical units = β thermal units

is valid for all the authors (history). Let us consider the arguments for the equation.

Mayer wrote

$Cp-Cv=Ph$

and calculated the mechanical value for a unit of heat.

Joule wrote in 1843

$(P1 - P2)\, h = C$

or a variant of this expression (Coelho 2009, p. 969). From 1845, a method unfolds which leads to an equation of the type

$Ph = C$.

Colding accepts the equation ‘α (mec.) = β (therm.)’ from the beginning: in his terms,

q (value of the mechanical effect) = ***q*** (value of heat appeared).

Thanks to his experimental data, he defended

$$\frac{\Delta F}{\Delta C} \approx 1$$

He did not present, however, a mechanical equivalent of a unit of heat.

Helmholtz neither wrote an equation as Mayer or Joule, but admitted the equivalence, that is to say, something of the form '*p mechanical units = 1 unit of heat*'. In 1847, he knew Joule's experimental work, accepted the concept of a mechanical equivalent of heat and used it.

On the conceptual or theoretical justification of the equations
Mayer came to that equation based on the statement "cause equals effect" (history). The 'cause' and the 'effect' in phenomena are called forces. According to his concept, forces are indestructible in quantity and transformable in quality. As the quantity of force, which is the cause, is equal to the quantity of force, which is effect, the equation

Cp-Cv (cause) = Ph (effect)

is justified. The sides of this equation are very different from a phenomenological point of view: heat and motion. This does not matter, since the transformability of force is admitted. Mayer pointed out, however, that 'transformation' of heat into mechanical power does not explain what is going on in the physical processes.
In sum, Mayer did not find an entity which is transformable and cannot be destroyed but rather a methodology of dealing with phenomena. He verifies if a cause-effect relationship can be applied to a certain phenomenon and establishes a numerical relationship between observable data. Thus, he connected domains, which were until then separated, such as motion and heat.

For Joule,heat is a kind of motion. Therefore, what we have in the equation are the magnitudes concerning the motion of weights and another kind of motion, which is heat. The equations served to determine the factor of conversion: how many units of mechanical power correspond to a thermal unit.
In sum, the thesis heat is motion renders homogeneous what, by observation, is diverse. Once this homogeneity is accepted, the equation is considered as an expression of conversion.

Colding achieved the homogeneity between heat and motion through the concept of 'force of nature': both motion and heat are forces of nature. The diversity of the parts of the equation, which is obvious through observation, is justified by transformation, motion is transformed into heat.

Helmholtz defended two ultimate forces, which influence each other reciprocally, i.e., if the quantity of one of them increases, the quantity of the other decreases. Hence, the quantity of force is conserved. Thus, when motion is made and heat is caused, the conservation of force demands that what is done in one domain appears in the other. *Conditio sine qua non* of conservation is that heat is a kind of motion. This is necessary so that heat can be conceived of through forces of motion and of tension.

In summary, we have the following.
1. In all the authors there is an element which unifies the parts of the equation,

α mechanical units = β thermal units,

- in Mayer,they are forces;
- for Joule,they are motion;
- in Colding, they are forces of nature;
- for Helmholtz, they are expressions of ultimate forces.

2. The observables referred to by the right and left-hand side of the equation are different, but the authors have an element to overcome the phenomenological diversity:

- Mayer's transformation;
- Joule's conversion;
- Colding's transformation;
- Helmholtz's correlation between the ultimate forces.

Heat and motion became homogeneous thanks to previous knowledge. This means the following. 'Heat', 'motion' or 'falling force' are forces in Mayer, because they appear as parts of the cause-effect relationship. When the author puts 'degrees of heat = mechanical units', he does it because the cause is equal to the effect. Mayer is, therefore, applying previous knowledge to phenomena and not inferring from phenomena. From 1845, 'nothing comes from nothing' became the basis for the dealing with phenomena. This statement justifies that equation. Once again, Mayer starts from previous knowledge, which is applied to phenomena. 'Force is indestructible' does not result, therefore, from an analysis of force but is instead a demand of the basic statements 'cause=effect' or 'nothing comes from nothing'. Transformation serves to link what by observation is diverse, as we have seen. Reuniting the data, it can be said that Mayer discovered a methodology to deal with phenomena. Based on observation, he established equivalences between domains which, until then, were not related in a precise manner. As he said in 1851, what is essential is the number, the equivalent between heat and motion. It could be added that what he did with the other domains, beyond mechanics and thermals, is similar. It is establishing relationships between what is observable, which permits him to generalise that procedure to the electrophorus, to plant growth, to animal motion, to the renewal of solar heat, etc.

Joule started from knowledge that came from the science of his time: heat is either a substance or motion. To be a substance implies from the experimental point of view, to be constant in quantity. In 1843, Joule showed that heat can be generated and destroyed by motion. Consequently, by the initial alternative, heat is motion. This conclusion about heat came, therefore, from the knowledge and methodology of his time. Once heat was reduced to motion, there was no difficulty with the connection heat-mechanical power: that is a conversion from a certain kind of motion into another kind. Beyond this, there is a factor of conversion obtained by experience, which reinforces the conversion of motion into motion.

Colding starts from the principle that forces of nature are conserved. This is an idea he had, according to him. When the idea was applied to motion and heat, he was led to admitting that motion had given heat in the same quantity. Something different would contradict the starting point. The transformation that he defends for forces of nature is not the result of research on the disappearing force. The concept plays however the role of connecting what is diverse from a phenomenological point of view. The fact that heat and motion are diverse is not an objection to the conservation of forces of nature, if transformation is admitted. The very designation of 'force of nature' induces one to admit extraordinary properties, such as indestructibility and transformability.

Helmholtz starts from the principle that two ultimate forces in nature exist. This is not information gathered from the study of phenomena. The justification comes from the theory of knowledge and from his philosophy of science. The task of physics is important here, since when, for example, the question arises of why not admit that in the falling of bodies, there exist ultimate forces of invisible particles, the reply can be, ultimate forces have already been found, by which the task of physics would be accomplished. On the other hand, that task justifies that ultimate forces are imagined, if they are not observed. The ultimate forces lead Helmholtz to consider the phenomena from the following point of view: there is something at rest, but tending to produce movement, or there is something in motion. As when phenomena are observed, either there is rest or relative motion, Helmholtz's categories permit observation to be subsumed. In the cases in which observation is not possible, the same categories hold, since we cannot imagine more than two states: motion or rest.

The dilemma of whether heat is substance or motion was known by these authors.

Their position, however, is diverse.

Mayer did not take a position on the dilemma. He said that heat is transformed into motion but noted that he did not defend that heat is motion. In fact, he does not need the thesis heat is motion, because the relationship is made through what is observed and diversity justified by transformation. Therefore, as the relationship between motion and heat does not occur through the nature of heat, it is not necessary for him to take a position on the substance or motion dilemma. (The electrophorus gives us an example of this dealing with phenomena. What Mayer developed only needed two observable elements: electric effects and mechanical action. Thus, it was not necessary to discuss the nature of electricity, as it was usual at that time.)

In contrast to Mayer, Joule needed the heat-motion thesis. This thesis was the alternative to the heat-substance thesis within the framework of the science of that time. The heat-substance thesis was an epistemological obstacle, since it was admitted that a motion could neither produce a substance nor be produced by its reduction in quantity.

Colding did not defend conversion but rather transformation. He did not unify heat and motion by motion but by a concept that subsumes both: 'forces of nature'.

Helmholtz's theory depends on motion. If heat is not motion, the thesis of heat conservation ceases to be valid, because conservation of force, in which one is a force of motion and the other tends to cause motion, cannot be spoken of, if heat is not thought of as a kind of motion.

Conclusion

Reuniting data and having in view the question of what energy is, it can be said that transformation and conversion do not result from an analysis of processes, as the terms by themselves could denote. Motion, of which heat would consist, was never an object of observation. Heat-force is the result of the theory. Concerning the question of what the authors did: Mayer developed a methodology of dealing with phenomena and elaborated a theory which leads to acts in conformity with the proposed method; Joule found some methods of measurement of the mechanical equivalent of heat; Helmholtz suggested a theoretical procedure based on the universal categories of rest and motion.

Energy

Capacity of doing work

Thomson introduced the term energy, which etymologically means activity. As heat was admitted by him (1851) as a kind motion and there was a conversion factor, the activity of a body concerning heat emitted and resistances overcome could be given mechanically. This was the first meaning of the concept: mechanical energy of a body (history).

Thomson's concept of 'mechanical energy' was replaced by 'energy'. The idea, a body does something mechanically was, however, maintained in the definition: energy is the capacity of doing work. Maxwell 1873 for instance writes: "the energy of a body may be defined as the capacity which it has of doing work" (p. 90).

This definition was criticised by Lodge, who instead proposed to understand energy as the work done on a body. The conservation of energy is then expressed in the form: "energy is neither produced nor destroyed, but is simply transferred" (p. 279).

Energy becomes a substance

According to Poynting, the energy existing in a space flows into the conductor through the lines that are perpendicular to the lines of the electrical and magnetic field. Based on this paper, Lodge defended the thesis: "The energy may be watched at every instant. Its existence is continuous; it possesses identity" (p. 483). Energy became something substantial. This is also expressed in a comparison with the usual theory:

"The fact is the stone never had any potential energy, no rigid body can have any; the gravitation medium had it however, and kept on transferring it to the stone all the time it was descending" (p. 486).

According to Planck, energy had been understood as a substance. He saw, however, a difficulty with this concept: it is impossible to localise the energy in the system. Energy, as a substance, is then considered by him as a concept which one day should be overcome.

Hertz criticised the concept of energy as a substance: this is not logically permissible, since energy has properties which contradict the concept of substance.

On contemporary topics on energy

The principle of conservation of energy and the concept of energy are often presented in the form: energy can neither be created nor destroyed but only transformed (textbooks). If energy cannot be produced and if there is some, it cannot be annihilated, then it must be a real existing thing. If energy can only be transformed, it must be a real thing as well: so real that its form can change. The concept of energy as a substance is understandable, therefore.

Indestructibility and transformability have their historical reason in Mayer's theory. Mayer did not find an entity which is transformable and cannot be destroyed but rather a methodology of dealing with phenomena, as we have seen. Let us now make recourse to his methodology. We are then aware that equivalence between certain magnitudes is established by us. Thus, we do not need the 'indestructibility' of an entity to express that the quantity does not change. As equivalence is established between different domains, we do not need to suppose the 'transformability' of the same entity. Thus, we can dispense with the unknown entity whose properties justify our own methodology.

How to understand that there are many forms of energy and the universality of the principle

The establishment of equivalences has been carried out concerning a wide variety of phenomena. We cannot rule out the possibility of establishing new equivalences. Insofar as such phenomena as considered as phenomena of energy transformation, the domains involved in the equivalence are seen as forms of energy. Thus, the universality of the principle and the variety of the forms of energy is a consequence of the great applicability of Mayer's methodology (Coelho 2009, p. 979).

Notes

[1]"Der Zusammenhang, in welchem, wie wir gesehen haben, die Wärme mit der Bewegung steht, bezieht sich auf die Quantität, nicht auf die Qualität" (1851, p. 43).

[2]"Wenn hier eine Verwandlung der Wärme in mechanischen Effekt statuirt wird, so soll damit nur eine Thatsache ausgesprochen, die Verwandlung selbst aber keineswegs erklärt werden. Ein gegebenes Quantum Eis lässt sich in eine entsprechende Menge Wassers verwandeln; diese Thatsache steht fest da und unabhängig von unfruchtbaren Fragen über Wie und Warum und von gehaltlosen Speculationen über den letzten Grund der Aggregats-Zustände. Die ächte Wissenschaft begnügt sich mit positiver Erkenntniss und überlässt es willig dem Poëten und Naturphilosophen, die Auflösung ewiger Räthsel mit Hülfe der Phantasie zu versuchen" (1845, p. 10).

1.4 Energy-Teaching

Teaching

There has been much research on the subject energy: either on students' misunderstandings (Watts 1983, Duit 1986, Nicholls and Ogborn 1993, Cotignola *et al.* 2002, Barbosa and Borges, 2006, and many others) or on teaching methods in order to avoid misconceptions (Solomon 1983, Prideaux 1995, Trumper 1990, 1991, 1997). Explanations of energy in high-school and university textbooks have been criticised: Lehrman 1973, Sexl 1981, Duit 1981, Hicks 1983, Duit 1987, Bauman 1992, Chrisholm 1992, Cotignola *et. al.* 2002, Doménech *et al.*, 2007. Historical approaches to the subject have also been developed (Valente 1999, Greenslade 2002, Hecht 2003, Roche 2003, Coelho 2009). Let us move on to some of the problems pointed out in order to consider if they can be overcome.

Duit 1987 pointed out some inconveniences of the concept of energy as something quasi material, defended by some physicists. According to Beynon 1990, there is so much confusion with energy "because it is not treated as an abstract physical quantity but something *real*, just like a piece of cheese" (p. 315). Empirical educational research shows alternative ideas such as 'Energy is fuel' or 'Energy is stored within objects' (Ogborn 1993, p. 73, Prideaux 1995, p. 278). There is, however, a reason for that concept of energy.

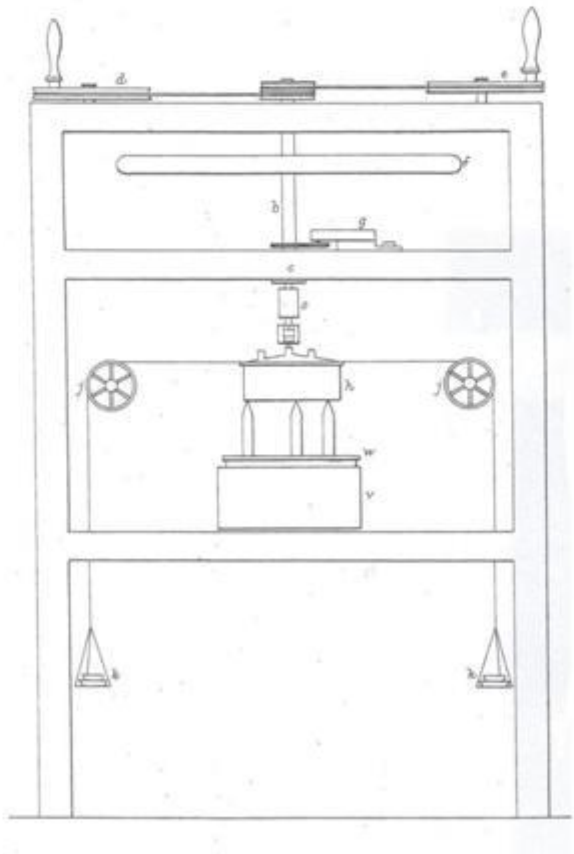
The most common presentation of energy in contemporary <u>textbooks</u> states: energy cannot be destroyed nor created but only transformed. If energy can be transformed, then forms of energy must exist. If they did not exist, then it would not make any sense to speak of the changes of energy form, which is the meaning of transformation. Connected with transformation appears the indestructibility of energy, which reinforces the idea of its reality. Thus, it is understandable that some textbooks present energy as something quasi material, as Duit stressed, and students think of it as something real. If energy is thought of as something real, then it is difficult for a student to articulate 'transformation of energy' with his observation of a falling body. A technical registration of a falling body yields no more information than the places and times of it. Thanks to this, a student can neither observe any cause of falling, as weight, nor the transformation of anything, as energy as something real. The concept of energy based on Mayer's and Joule's dealing with the phenomena (<u>philosophical part</u>) enables us to overcome that difficulty. In fact, Mayer had already pointed out that 'transformation from falling force into motion' cannot express anything but a numerical relation between both (1851, p. 41-2).

Textbooks published towards the end of the nineteenth century or the beginning of the following century used the expression "principle of equivalence" concerning the heat-motion relationship (Verdet's *Thermodynamics*, 1868, Poincaré's *Thermodynamics*, 1892, Preston's *Theory of*

Heat, 1919). Müller and Pouillet's *Physics*, 11th edition, 1926 pointed out further that with the thesis 'energy is indestructible' the principle is not an experimental law anymore but a postulate[1].
Nowadays, we use the postulate form but we have difficulty in defining energy. Understanding the principle as a principle of equivalence and the equivalence factor as a value, which is determined experimentally, the difficulty in the explanation does not seem to have existed in teaching thermodynamics.

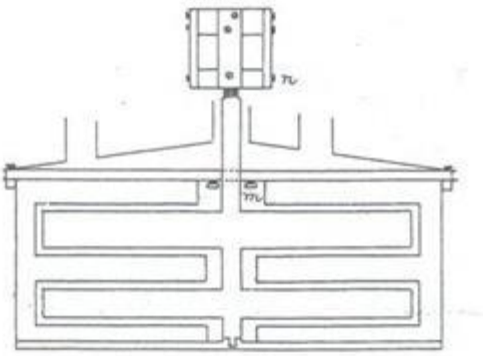
On *Tipler's schema of Joule's paddle-wheel experiment* (see Tipler 2000, p. 554)
From a historical point of view, this is not Joule's schema.
From a physical point a view, such a paddle-wheel would not lead to the value of the mechanical equivalent of heat.
From a pedagogical point of view, it would be useful to give the idea of friction for different reasons. (See also Steam, Work, Energy).
Joule's experiment is explained as follows: mechanical work is converted into heat (Arons 1999, p. 1065) or the potential energy is transformed into heat (Bergmann and Schaefer 1998, p. 1032, Halliday and Resnick 1993, p. 614). As the friction, which could increase the temperature of the water in a reasonable way, is not shown, a student will accept that there was a transformation of energy, but will not understand the reason for it. This can lead to the idea of energy as something with somewhat strange properties.

In 1878, Joule took up the experimental configuration of 1850 again with some improvements



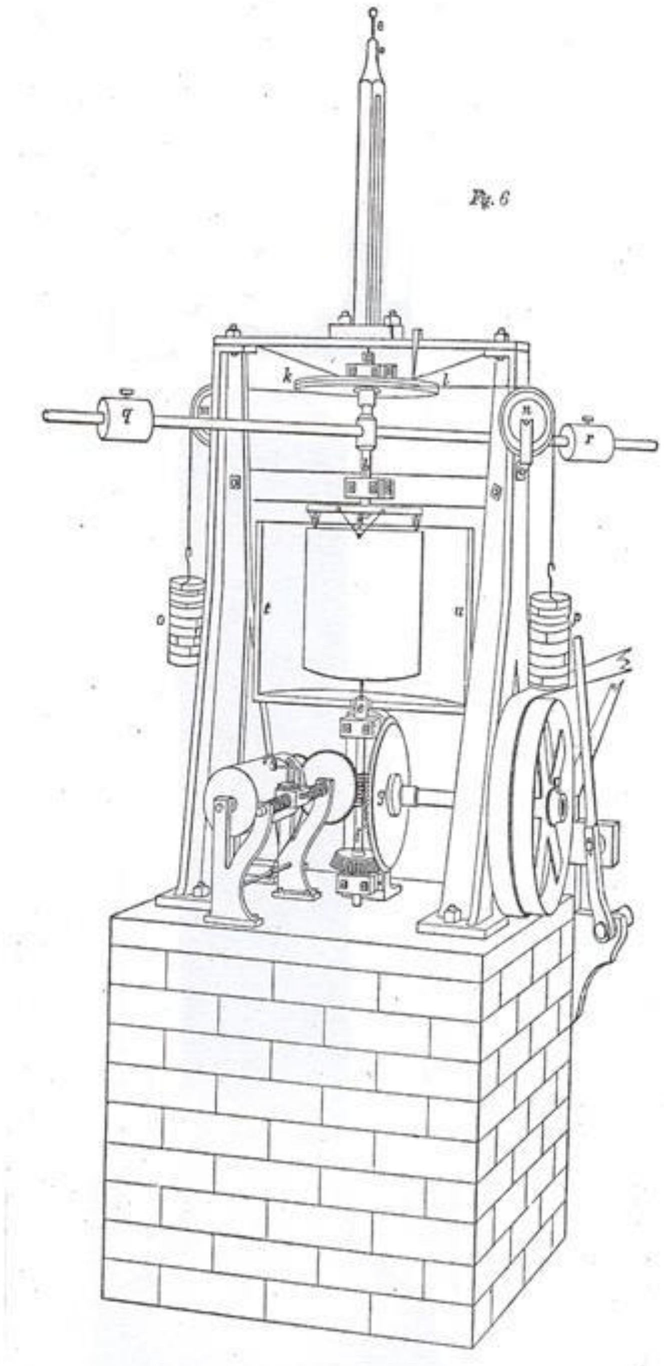
From Joule's paper

The schema of the section of the calorimeter is not very different from the first one.



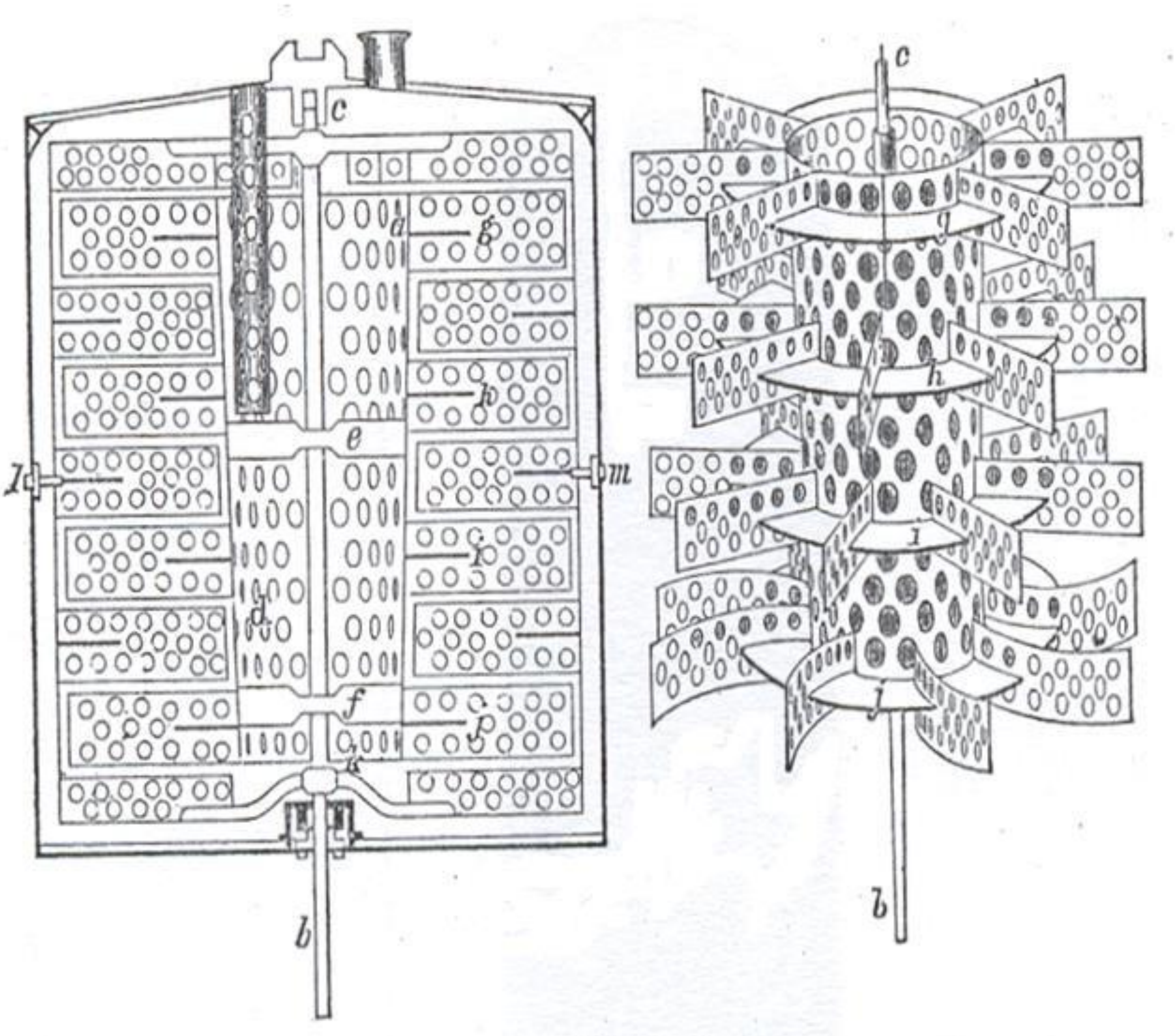
From Joule's paper

In 1879, Rowland carried out an analogous experiment, which can be useful to show in the classroom.



A perspective view of the apparatus, from Rowland's paper

The following pictures show the section of the calorimeter (left) and a perspective view of the revolving paddles removed from the apparatus (right):



This picture clearly shows the means of friction. It contrasts with the contemporary schemes for the paddle-wheel experiment (see Young and Freedman 2004, p. 653 or Serway's *Physics* 2006).

Let us move on to the interpretation of this experiment thanks to Mayer's methodology. Mayer established *equivalences* between different domains, such as motion and heat. In using this approach to the previous experiment, we are aware that we establish equivalence between certain magnitudes: weight and height on one side and units of heat on the other. Thanks to this, an equivalent between mechanical magnitudes and heat was determined. The equivalent holds in analogous situations. Thus, we express what we did without the need of a real something which is 'indestructible' and 'transformable'. Thus, using the concept of equivalence, we can understand our dealing with the phenomena and dispense the real something, which is a result of a semantic development

of the concept of energy.
Duit 1987, Prideaux 1995, Arons 1999 corroborate Feynman's concept of energy: energy is not a concrete thing. They defend, moreover, that energy conservation is a mathematical principle. This thesis agrees with the idea of equivalence (see Coelho 2009). Whereas, however, Feynman's concept is too abstract for teaching at a basic level (Duit 1987, p. 145), Mayer's dealing with the phenomena can be introduced without that shortcoming, according to the empirical research carried out.
Doménech *et al.* 2007 deconstruct ideas which could lead to an interpretation of energy as something possessed by the objects themselves (p. 51-53). The historically based approach avoids the idea of energy as a real entity, since it does not lead to it.
If energy is a substance and the transformation of energy is admitted, then the extremes of the transformation must be forms of that something which is in transformation. This justifies the thesis that heat is a form of energy. This thesis has however been criticised (Cotignola *et. al* 2002, p. 285, Doménech *et al.* 2007, p. 54). The concept of equivalence avoids the transformation of a real something and, consequently, the criticism of it.
Thomson's concept of energy in the 1850s presupposes Joule's thesis 'heat is motion' (see also Temperature - what can be find out when we measure it?). In fact, Thomson defined. In the course of time, Thomson's expression "mechanical energy of a body" was reduced to 'energy of a body' (history). This concept included, however, the first idea, since energy was defined as the 'capacity of doing work'. As the concept of energy was worked out in a mechanical context, the criticism that that energy definition is too restricted is understandable (Lehrman 1973, Sexl 1981, p. 287, Duit, 1981, p. 293, Hicks 1983, Kemp 1984, p. 234, Doménech *et al.* 2007, p. 49).

Notes

[1] "*Energie (beliebiger Form) kann weder erzeugt, noch vernichtet werden*. Die einfachste Gestalt nimmt das Energieprinzip wohl in der Form an: *Die Summe aller einem abgeschlossenen System innewohnenden Energieformen bleibt bei sämtlichen Umwandlungen desselben konstant*.
Bei noch etwas schärferer Fassung nimmt der vorangehende Gedankengang folgende Gestalt an: Energie wird als unzerstörbar angesehen. Das Energieprinzip ist somit zunächst kein empirisches Gesetz, sondern ein *Postulat*, das sich allerdings mit den Erfahrungstatsachen (Äquivalenzgesetz) durchaus im Einklang befindet" (1926, p. 126).

Concepts of Force

The most common definition in contemporary textbooks asserts that force is the cause of acceleration or, in other words, force is exterior to the body, acts on it and changes its natural motion. Several objections have been raised against this concept (Hamel, Russell, Platrier, Ludwig, Wilczek). In fact, the difficulty with concept of force is not a new one. D'Alembert, Lazare Carnot, Saint-Venant, Kirchhoff, Hertz, did not only criticize the most common definition of force but also developed new theories in order to avoid the concept of force as the cause of acceleration. As force is a fundamental concept and basic in physics, it is not surprising that force is the dominant theme in the misconceptions' literature (Carson & Rowlands 2005, p. 473).

History

Descartes was the first to create a theory of motion based in two main laws: the first indicates the states, which a body maintains by itself; the second, how these change. According to his theory, changes in motion occur by impact. The first law of nature tells us that each thing remains in the state that it is, if nothing changes it. The states are: resting, moving uniformly and the form of the body. The second law tells us that every body that moves tends to continue its movement in a straight line (see also History of Understanding of Motion). The third law deals with changing of motion. It states that if a moving body meets another stronger than it, it loses nothing of its motion; and if it meets one that is weaker than it, it moves the other with it and loses as much motion as it gives.
This law tells us what happens if the moving body is the weakest and if it is the strongest. Nothing is explicitly said about the case where the bodies are equally strong. However, this can be deduced.
If the impacting bodies are named B and C, where B is the moving body, the third law can be given in the following form:

1. if B is weaker than C, B rebounds after the impact;
2. if B is stronger than C, B drives C forward.

From this, we can conclude that if B is as strong as C, then B cannot only rebound or only drive C forward, but must rebound and drive C forward. In order to decide which body is the strongest, we have to compare their forces. If the two bodies are in motion, we compare the product of mass and velocity of both bodies:

$$\frac{M_B V_B}{Mc V_C}.$$

If B moves and C is at rest, then we compare only their masses: $\frac{M_B}{Mc}$.

The reason for this lies in the first law: a body maintains its state of rest or of moving uniformly means that the state of motion is not ‘more important’ than staying at rest. Thus, as these states are neutral, the comparison of the forces of these bodies depends only on their masses. Once the force of impacting bodies is determined, the results of impact can be calculated in two steps. Firstly, the quantity of motion which body B can transfer to C must be calculated. The equation for this is the following

$$Q_t = \frac{Mc}{M_B + Mc}(V_B - V_C).$$

Secondly, the final velocity of B and C are calculated by the following equations:

1. If the force of B is greater than that of C, the transferable quantity is completely transferred and the final velocities V'B et V'C are calculated by

$$V'_B = V_B - Q_t$$

$$V'_C = V_C + \frac{M_B}{Mc}Q_t;$$

2. If the bodies are equally strong, the transferable quantity must be divided equally between the two bodies, that is to say, Qt is divided by 2, thus

$$V'_B = V_B - \frac{Q_t}{2}$$

$$V'_C = V_C + \frac{M_B}{Mc}\frac{Q_t}{2};$$

3. If the moving body is the weaker, the transferable quantity is not transferred. Thus, making Qt=0 in the last two equations, it follows that

$$V'_B = V_B$$

$$V'_C = V_C.$$

These equations are not indicated by Descartes but instead inferred from the results of impacts presented in the rules of impact. This means that through them we can calculate the final results of all impact rules. In the following table all Descartes' rules are indicated (Pr. II, § 46-52) (see also Cartesian Theory of Motion)

Rules of Impact

--- **Before impact** ----------------------------------- **after impact** -----------------------------------

Rule	Mass	Velocity	Direction	Direction	Velocity
R 1	B = C	$V_B = V_C$	→ ←	← →	$V'_B = V'_C$
R 2	B > C	$V_B = V_C$	→ ←	→ →	$V'_B = V'_C$
R 3	B = C	$V_B = 6$ $V_C = 4$	→ ←	→ →	$V'_B = 5$ $V'_C = 5$
R 4	B = ½ C	$V_B = 3$	→ . (rest)	← . (rest)	$V'_B = 3$
R 5	B = 2 C	$V_B = 3$	→ . (rest)	→ →	$V'_B = 2$ $V'_C = 2$

R 6	B = C	$V_B = 4$	→. (rest)	←→	$V'_B = 3$ $V'_C = 1$
R 7_a	B = ½ C	$V_B < 2V_C$	→→	←→	$V'_B = V_B$ $V'_C = V_C$
R 7_b	B = ½ C	$V_B = 5$ $V_C = 2$	→→	→→	$V'_B = 3$ $V'_C = 3$
R 7_c	B = ½ C	$V_B = 2V_C$	→→	←→	

Let us take R 6 as an example. Body C is at rest. Then the comparison of the force of B and C depends on their masses. As the masses are equal, the bodies have the same force. Thus, we have to use the equations

$$V'_B = V_B - \frac{Q_t}{2}$$

$$V'_C = V_C + \frac{M_B}{M_C}\frac{Q_t}{2};$$

The transferable quantity from B to C is

$$Q_t = \frac{M_C}{M_B + M_C}(V_B - V_C) = \frac{1}{1+1}(4 - 0) = 2$$

Thus,

$$V'_B = V_B - \frac{Q_t}{2} = 4 - \frac{2}{2} = 3$$

$$V'_C = V_C + \frac{M_B}{M_C}\frac{Q_t}{2} = \frac{1}{1}\frac{2}{2} = 1$$

These are the results predicted by Descartes for the 6th rule.

In the third case of the R 7, Descartes does not give any numerical example but now, it can be calculated. If MB=1, MC=2, VB=4, VC=2 is taken, it follows that

$$V'_B = V_B - \frac{Q_t}{2} = 4 - \frac{\frac{4}{3}}{2} = \frac{10}{3}$$

$$V'_C = V_C + \frac{M_B}{M_C}\frac{Q_t}{2} = 2 + \frac{1}{2}\frac{\frac{4}{3}}{2} = \frac{7}{3}$$

As the velocity of B is greater than that of C, B must rebound; and as the final velocity of C is greater than the initial velocity, it follows that C is thrust forward by B. This result is conform to Descartes' explanation for this case: B rebounds and C is thrust forward.

In 1687, Newton published the *Mathematical Principles of Natural Philosophy*. The fundaments of his theory consist of eight definitions and three axioms.

Definitions

Def 1	**Quantity** of matter
Def 2	**Quantity** of motion
Def 3	**Inherent** force
Def 4	**Impressed** force
Def 5	**Centripetal** force
Def 6	Absolute **quantity** of centripetal force
Def 7	Accelerative **quantity** of centripetal force
Def 8	Motive q**uantity** of centripetal force

The first definition concerns matter, the second motion and the other six concern forces. Five of the eight propositions define quantities and the other three, concepts: inherent, impressed and centripetal force. Centripetal force is a particular case of impressed force[i]. Thus, there are two kinds of force:

inherent and impressed. From the first kind there is only one force: the force of inertia. Centripetal, pressure and impact are examples of impressed forces. Force of inertia justifies that a body resists change of its motion or resting. Changes in motion require impressed forces. With these two kinds of force the axioms are connected.

Axioms

1st Axiom	Free body	Constant velocity
2nd Axiom	Non-free body	Non-constant velocity
3rd Axiom	Non-free bodies	Constant velocity

According to the first law of motion, a body perseveres in its state of resting or of moving uniformly in a straight line, unless an impressed force constrains it to change its state[ii].
The second law of motion states:"The change of motion is proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed“[iii]. By “motion” is understood ‘quantity of motion’, i. e., the product of mass and velocity of the body (Definition II).
A force is double another one, Newton adds, if the change caused by the first is double the change caused by the second[iv].

Euler’s *Mechanics or the Science of Motion Presented Analytically*, 1736, consists of two books: the first deals with free motions and the second with constrained motions. Euler’s approach to free motion is based on the following sequence: a body by itself stays at rest or maintains the uniformity and rectilinearity of motion. Force is that which changes these states[v]. In conformity with this, Euler carries out the decomposition of force. If a body is moving on a plane, two components of force are considered: tangential force, whose effect is only the change of velocity[vi], and radial force, which has no other effect than the change of direction of the motion[vii]. Euler’s approach can be interpreted in the following way (Coelho 2010, p. 93-94). The mechanical states of a body by itself correspond to the motion of reference. Force is a deviation from this motion. The components of force correspond to the negations of the characteristics of the motion of reference. Let us express this more clearly. The characteristics of the motion of reference are: the path is

rectilinear and the **velocity** is **constant**.

The deviation from this motion corresponds to the

negation of **rectilinear** and **constant velocity**.

Logically this is

non**-rectilinear** or non**-constant velocity.**

The components of force are connected with this

non-rectilinear - radial component
non-constant velocity - tangential component

Let us move on to constrained motion.

Euler was the first to deal with the motion constrained by a surface. In this case, three components of force are considered. The first one concerns the pressure exerted by a surface upon a moving body. This component must exist if the motion is conditioned. The other two components can exist or not. If they do not exist, a body covers the shortest line on the surface and moves uniformly, says Euler. If a body does not move uniformly, then there is a component tangential to the motion. If the body does not cover the shortest line, another component is considered[viii]. Euler's approach can be interpreted as follows. The motion of a body constrained by a surface – it covers the shortest line uniformly – represents the motion of reference under those circumstances. Force is a deviation from this motion (Coelho 2010, 105-106). The components of force correspond to the negation of the characteristics of this motion of reference. Let us express this more clearly.

The characteristics of the motion of reference of motions constrained by a surface are: the path is

the shortest line and the **velocity** is **constant**.

The deviation from this motion corresponds to the

negation of **the shortest line** and **constant velocity**.

Logically this is

non**-shortest line** or non**-constant velocity.**

The components of force are connected with this

non-rectilinear - the 2nd component
non-constant velocity - the 3rd or tangential component

The **1st component** stems from the surface. It makes the difference between a constrained and a non-constrained motion. In the domain of motions constrained by a surface, the motion of reference then holds, with those characteristics.

Euler dealt with the question of how to connect the concept of force with phenomena. According to him, it is difficult to think of force without motion. Otherwise, motion can exist without force. Hence, he concludes that all forces which we observe, have origin in motions[ix]. This difficulty with observing force became the problem of the concept.

A Digression

Let us admit, as proposed by Euler, that all forces have origin in motions. In this, it is presupposed that these motions are not observable. To carry out an explanation conform to that admission, motions have to be imagined in such a way that their consequences have the effects which are ascribed to forces. This exercise of scientific imagination aims to overcome a difficulty which stemmed from the concept of force as the cause of acceleration.

In 1743, d'Alembert published the *Traité de Dynamique*, whose first part deals with the principles of mechanics. The first principle states that a body maintains its rest or if moving, will move rectilinearly and uniformly, if no external causes act on it. As accelerated motions are observable, there can be no doubt concerning the existence of those causes[x]. There was, however, an objection against the use of force in mechanics.

The cause of motion was represented at that time by f in the equation $f\,dt = du$, where dt and du represent small quantities of time and velocity. According to him, the thesis that force is the cause of acceleration is based on the "vague" and "obscure" principle that the cause is proportional to the effect[xi]. In fact, he continues, excepting impact, force is unknown to us[xii]. Hence, he carried out a theory of mechanics without supposing knowledge of the nature of force. In sum, d'Alembert admitted force from an ontological point of view but not as an object of knowledge due to the lack of observability.

In 1750, Euler published an article with the title "Discovery of a New Principle of Mechanics". The new principle is only an equation, whose form is '***force =mass·acceleration***'. Force is decomposed into three components, symbolized by Px, Py and Pz. With this kind of decomposition, the information concerning the path covered by a body and how it is covered lies in the coordinates and in the acceleration along the coordinates.

Lazare Carnot developed a new theory of mechanics in order to avoid the concept of force as the cause of acceleration. According to his *Principes fondamentaux du mouvement et du repos*, 1803, there are two ways of carrying out mechanics: either as a theory of force or as a theory of motion. The first one, which was followed by almost all the authors, has one shortcoming, being based on the "metaphysical" concept of force. This gave him the reason for opting for the second method[xv]. The problem pointed out by Carnot concerns the observation of force and was presented in considering machines. Some machines from that time were based on man and animal power. If a human being brings a machine into motion, he is the cause of that motion. The cause of motion was force, according to mechanics. Is this force, Carnot questioned, the structure of the skeleton of the human being or of an animal or their wills? Does a double force mean, he continued to ask, that a human's or an animal's will in the first case is double that in the other?[xvi]

Carnot proposed then to identify force with the quantity of motion which a force caused in a body[xvii]. In doing this, we do not know more about the force which causes motion, called “first cause”, but it does not disturb the theory. The “first cause” is unobservable; but within the theory force is certain quantity of motion.

Barré de Saint-Venant also carried out a reorganization of mechanics in order to avoid the concept of force as cause of acceleration. Barré’s theory, presented in his *Principles of Mechanics*, 1851, starts from a unique proposition, which states that the acceleration of bodies depends on the points which constitute them. For the measurement of mass, the reciprocal alteration of the velocities of bodies by impact is proposed[xix]. As the use of this measurement process is very difficult, Saint-Venant recommended measuring mass by weighing[xx]. Force is defined as the product of mass by acceleration[xxi].
Barré aimed to overcome the difficulties with the concept of force in making force a mere mathematical concept. However, other difficulties arose: not only concerning the process of measurement of mass but also the interpretation of phenomena. The terms used in the dealing with phenomena remind us of the traditional concept of force: ‘acting forces’, ‘force acts on bodies’, and analogous expressions[xxii]. Saint-Venant was aware of this.

In the following year, Reech published the *Cours de Mécanique*, whose aim was, however, very different. Criticised by him is the ‘relative concept of force’. According to this concept, force is understood through its ‘evident geometric effect’, i. e., by the deviation from a certain motion. Instead, Reech proposed the ‘real, absolute concept of force’. This reality is connected with the sensation of our muscles.
Our sensation, says Reech, awakened in us the idea of a certain quantity, called pressure or traction, which is the cause of the alteration of the motions of bodies which have been touched. This is the true idea of force that we should have[xxiv]. Force is then defined, he continues, as pressures or tractions that we can make through our organs on the bodies surrounding us[xxv]. The process of measurement proposed for force reflects the definition. Force is to be measured through a convenient thread, whose changes in length indicate the magnitude of force[xxvi].
This process of measurement involves some difficulties. As Reech pointed out that a thread has some mass and this influenced the measurement of force. Another difficulty concerns the limitations in using a thread to measure force, as, for instance, in the case of celestial motions. For such cases, he proposes measuring force thanks to the deviation from a certain, conventional motion. To play this role, he chooses the motion of the law of inertia, since it is the simplest motion.

The preface to Kirchhoff’s *Mechanics*, 1876, announces a restructuring of mechanics, whose leitmotiv lies in the concept of force. Disagreements among physicists concerning the law of inertia and the parallelogram of forces lie in the concept of force: in the lack of clarity of ‘cause’ motion and ‘tendency to

cause' motion[xxvii]. To avoid these problems, Kirchhoff restricts the function of the mechanics to the description of motion[xxviii].
The primitive notions of Kirchhoff's mechanics are space, time and matter; force and mass are to be constructed within the theory[xxix]. This plan was, however, not carried out successfully. Kirchhoff himself detected that there are forces in mechanics which cannot be subsumed by his theory.[xxx] Another difficulty concerns the interpretation of phenomena. His terminology leads us to think of force as a cause: 'forces act', 'acting forces', 'forces are exerted', 'exerting forces'[xxxi].

Mach´s *Mechanics* published in 1883, presented a solution for the concept of force, which was taken up from a short paper written in 1868. The starting proposition of Mach's proposal states that bodies in interaction cause reciprocal acceleration[xxxii]. This is considered a matter of fact. Taking one body as a unit, Mach continues, the mass of the other is measured through the proportion of the accelerations due to the interaction of both bodies[xxxiii]. Force is then defined as the product of mass and acceleration[xxxiv]. Even though Mach gave a new definition of force and proposed to understand it as a mere theoretical concept, the approach to phenomena was marked by the traditional interpretation. Force is any circumstance of which the consequence is motion, says Mach[xxxv]. This leads to the idea that force is the cause of motion.

Hertz in his posthumous work, *The Principles of Mechanics*, pointed out some difficulties concerning the concept of force. If we swing a stone tied to a piece of string in a circle, exemplifies Hertz, we are conscious of exerting a force upon the stone. This agrees with the definition of force: force is independent of motion and the cause of it. Newton's third law, he continues, requires, however, an opposing force to the force exerted by the hand upon the stone. Here the problem begins: "In our laws of motion, force was a cause of motion, and was present *before* motion. Can we, without confusing our ideas, suddenly begin to speak of forces which arise through motion, which are a consequence of motion?" (1899, p. 6). Since force is defined as the cause of motion and there is a force that is a consequence of motion, the theory is not logically permissible, according to Hertz.
His solution for force consists of a distinction between force in thought, which results from an imagined system, and force in practice, which is a measurement process. Between the constructed concept of force and the methods of measuring force there is only a mere correspondence.[xxxvi]

According to Poincaré, 1897, "to say that force is the cause of acceleration is to do metaphysics"[xxxvii]. He defends instead that a concept of force should be worked out from its measurement process. Hence, he began to consider the definition of equal forces: two forces are said to be equal if they attain equilibrium or produce the same acceleration on the same mass. Poincaré comments: we cannot connect and disconnect forces to or from bodies as horses to coaches or engines to carriages. It was said as well that two forces are

equal if they balance with the same weight. Poincaré pointed out that the weight depends on the place. Furthermore, Newton's third law is employed, in such cases, as a definition and not an experimental law.
From his analysis of the processes of measurement of force and mass, he concludes that it is impossible to give a satisfactory idea of mass and force within classical mechanics[xxxviii].

Contemporary textbooks: 20th and 21st century

1. Force-cause
Force is in general characterized as follows: force is exterior to the body, acts on it and changes its state of resting or moving rectilinearly and uniformly. This typical characterization has been expressed in different ways.
Webster 1904:
"The property of persistence thus defined is called *Inertia*. This gives a criterion for finding whether a force is acting on a body or not [...] Force **is acting on** a body when its **motion is not uniform**" (p. 21).

Feynman 1974:
"The Second Law gave a specific way of determining how the velocity changes under different influences called *forces*"; and "If an object **is accelerating**, some **agency is at work**" (§ 9-4).

Wolfson and Pasachoff, 1990:
"Why are we so interested in knowing about forces? Because **forces cause changes in motion**" (p. 76).

Knudsen and Hjorth 1996:
"Newton introduced the concept of *mechanical* ***force*** **as the cause of the acceleration** of an object. This causal description of the motion of an object constitutes what we call *dynamics*" (p. 29).

Sears and Zemansky 2004:
"the presence of a net **force acting on** a body **causes** the body **to accelerate**" (p. 129)

This kind of definition of force is presented in many textbooks (Voigt 1901, Lenard 1936, Sommerfeld 1947, Schaefer 1962, Budó 1974, Hestenes 1987, Alonso and Finn 1992, Daniel 1997, Gerthsen 2006, Kuypers 2008). There is a reason for this, which lies in the law of inertia.

The law states: **free body => constant velocity**.
It follows: **non-constant velocity => non-free body**.
Non-constant velocity means acceleration. Therefore, if the motion of a body is accelerated, it is a non-free body. Thus, something acts upon it and changes its natural motion. The authors, who define force through the properties of that something, understand force in conformity with the law of inertia. As most

physicists have admitted the law, it is understandable that they defend that concept of force.

2. **Force-cause variant: effort**
Some physicists defend a variant of the force-cause concept in understanding force as the effort felt by the pulling or pushing of an object. Planck 1916 says that the cause of the motion is called force and "it corresponds to the **effort**, which we feel, if that same motion had been produced through our muscles instead of the bodies, which caused it"[xxxix].

Nolting 2005:
"The concept of force can only be defined indirectly through its effects. If we want to modify the state of movement or the shape of a body, for example, using our muscles, then an effort will be necessary [...] **This effort is called force** [...] We observe everywhere in our environment changes in the states of motion of certain bodies [...] We see their causes equally in forces, which in the same way as our muscles, act on the bodies"[xl].

This kind of definition of force has its origin in Reech's theory, 1852. An important follower of his was Jules Andrade, who defended that mechanics is "essentially anthropomorphic"[xli]. Poincaré 1900 defended, however, that that notion of effort does not acquaint us with the true nature of force and that the anthropomorphism cannot provide the foundation of anything truly scientific or philosophical[xliii].

3. **The equation F=ma is sometimes used to define force**.
Fließbach 2007:
"Newton's second axiom embraces the following definitions and affirmations:
1. Definition of mass
2. Definition of force
3. [...]"[xliv]

The equation referred to as Newton's second axiom is $F=ma$. If this equation defines mass, then the mass definition is given by force and acceleration. As, however, force is defined by the same equation, what force is depends on what mass and acceleration are. Thus, we remain not knowing what both are. This kind of definition was criticized by Mach in 1868. More precisely, he criticized that weight was defined by mass and mass by weight. In this case, the equation W=mg was used to define mass and weight.

4. Force is a Thing of Thought
If force is the cause of a motion which a body could not have by itself, force must be a real existing thing. If not, it could not have any effect upon a body. However, some physicists defend that it is not a real existing thing. Hamel 1912:

"Force itself, however, we do *not* define as cause of motion, **force is a thing of thought** and not a natural phenomenon"[xlv].

Platrier 1954:
"In fact, **force is only a human concept** and we have no knowledge of the profound cause of motions"[xlvi].

Ludwig 1985 defends the thesis that the concept of force does not describe anything which exists in reality. In his own terminology, **force does not belong to "real text"**[xlvii].

The concept of force as cause of acceleration is logically based on the law of inertia. The law implies that acceleration is a sufficient condition for force. This means that what is observed is acceleration; force is inferred due to the law of inertia. This makes it understandable that authors who look for the force-cause in phenomena do not find it. In other words, what can be observed or experimentally reached is acceleration; some inferred force from it and other claim that they do not observe force.

5. We do not know what force is
Sometimes authors present force as the cause of acceleration and its measurement process but when faced with the question of what is force, they say that we cannot answer it.
Bergmann and Schaefer 1998:
"N*obody knows what energy is* [...]. We can neither say anything about what mass, force [] "in reality" are" (p. 135).
"As the equation [F=ma] states, force is the magnitude which an acceleration **a** to a mass *m* gives" (p. 97).
"This property of force [a mass to accelerate/of accelerating a mass] was used by us [...] to define force" (p. 107).

Dransfeld, Kienle and Kalvius 2001:
"Here we have to put forward the question 'What is Energy?' However, like the question of what force is, we cannot answer this." (p.109)
"A*s a consequence of the inertia of matter is therefore a force needed in order to change the velocity of a body*" (p. 71)

Fictitious Force
Let us consider a table which is accelerated with a ball on it. The ball moves in the direction contrary to the table's motion and is accelerated for an observer moving with the table (see Pohl 2004, p. 89).

As the ball's motion is accelerated, a force must be there. However, no force is acting upon it. We are, therefore, faced with the following situation: there is an accelerated motion and no action is exerted upon the accelerated body. As a body by itself cannot have an accelerated motion, a force must be admitted; as there is nothing exerting an action upon the body, this force is called “fictitious”. Let us suppose that instead of ‘fictitious force’, ‘no force is acting upon the moving body’ is admitted. In this case, a body could have an accelerated motion by itself. If a body could have both a uniform motion and an accelerated one by itself, the law of inertia would not hold.

Textbooks on Mechanics usually turn students’ attention to the point: fictitious forces have real effects (Daniel 1997, Tipler 2000, Dransfeld 2001, Fließbach 2003, Nolting 2005). This remark is understandable as the characterisation of force suggests the contrary. If something is fictitious, it is not real and being not real, it can do nothing. Hence, it cannot have any effect. As there is an exception to this, the remark is justified. The necessity of the remark shows, however, that the concept is not adequate in order to develop logical thinking. (See also History of Inertial Force)

Weight

There has been much research concerning weight, weightlessness and students’ difficulties with these. Galili 2001 uncovered three kinds of definitions of weight in physics textbooks: weight is a contact force acting downwards on the support; weight is a contact force exerted upwards on the body by the support; the weight of a body is the force, which acts downwards and causes spontaneous falling (p. 1082).

The concept, weight is the gravitational force exerted on a body, is called nominal definition of weight. The other definition, the operational one states that a body’s weight is the result of weighing. These two definitions can, however, cause some difficulties. Let us imagine the following situation.

A body is on a scale and the scale shows some weight. During the falling of both the body and the scale, the scale does not show any weight of the body. According to the operational definition, a student would say there is **no weight** by falling. As, however, the motion of the body is accelerated, he would say that **weight is acting** on the body. Thus, he faces the situation that through one way of thinking, there is weight and through the other, there is not. This difficulty can be overcome. However, if the thesis is accepted that the nominal definition of weight is the old definition and the operational one is the new one (History of Weight Concept), then that difficulty will disappear.

Philosophy

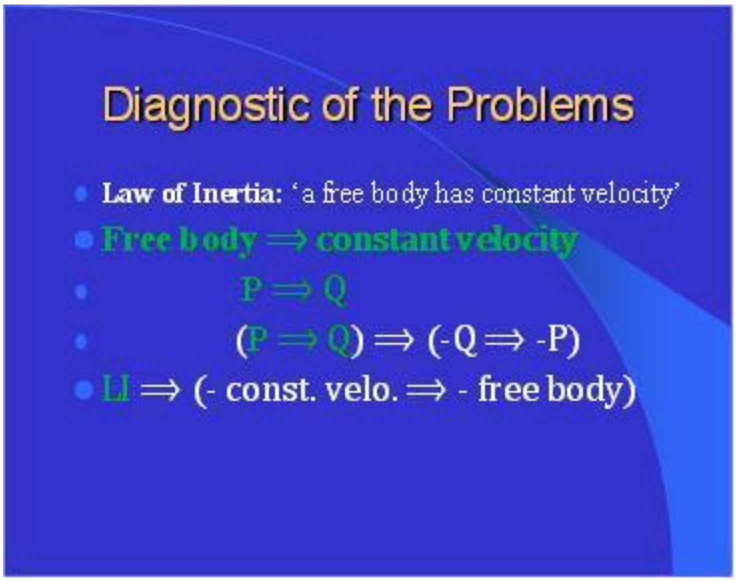
D’Alembert’s and Carnot’s difficulties concern the lack of observability of force. What they could observe were motions. Nevertheless, they admitted real forces. The admission of what is not observable can be understood thanks to the law of inertia. Both accepted this law as the first statement of their theories. Thus, if they had not admitted the existence of force, their theories would not have been logically consistent.

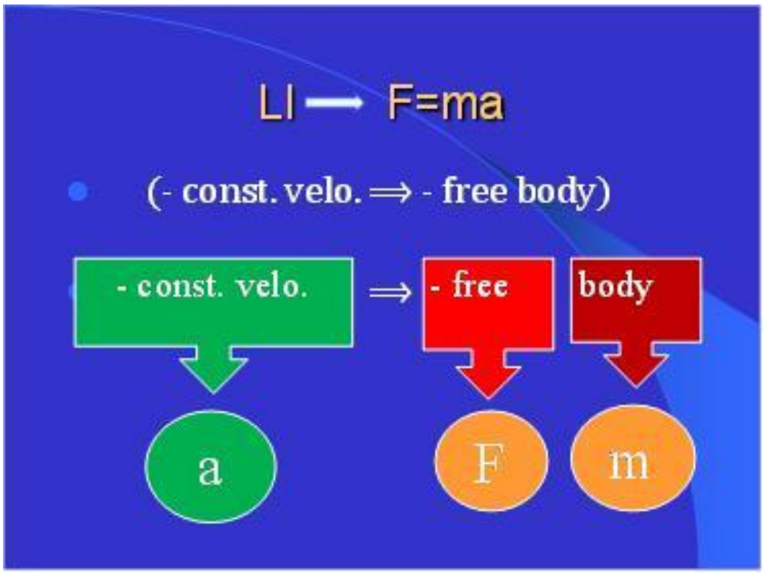
Saint-Venant's and Mach's theories of mechanics start from acceleration. In a second step, they define mass and finally force as a mere mathematical concept. That starting point of their theories is, however, incompatible with the classical system, for acceleration implies force, since the law of inertia is admitted. This difficulty can now be overcome.

Kirchhoff defined mechanics as the science of motion and planned to carry out a theory based on motion. He himself pointed out a difficulty: if there is a system of forces, it is not possible to determine the components only through motion. If force is understood as information drawn from some experiments, it follows that those components result from previous experiments. As these experiments are motions, Kirchhoff's plan is free of that difficulty.

Hertz's solution for force shows the difficulty in connecting force with phenomena. This is now clear: as we can observe only acceleration, force cannot be seen in phenomena.

Wilczek characterizes the assumptions concerning force as "a sort of folklore" (2005).





Motion of reference and force

Newton's force represents a deviation from the states referred to in the law of inertia. Euler conceived force as a deviation from a certain motion in creating the theory of the motion constrained by a surface. Reech criticized force because it was considered as a deviation from a certain motion. In Hertz's theory, there is force if there is a deviation from the motion of the fundamental law. This concept, force as a deviation from a certain motion, is also present in the decomposition of force.

This concept of force avoids the concept of force as the real cause of acceleration, which is a theoretical consequence of the law of inertia.

Notes

[i]"Est autem vis impressa diversarum originum, ut ex ictu, ex pressione, ex vi centripeta" (p. 2).

[ii]"Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus illud a viribus impressis cogitur statum suum mutare" (p. 13).

[iii]"Lex II. *Mutationem motus proportionalem esse vi motrici impressae, & fieri secundum lineam rectam qua vis illa imprimitur* " (p. 13).

[iv] "Si vis aliqua motum quemvis generet; dupla duplum, tripla triplum generabit, sive simul & semel, sive gradatim & successive impressa fuerit" (p. 13).

[v]"Potentia est vis corpus vel ex quiete in motum perducens vel motum eius alterans" (vol. I, § 99).

[vi]"Vis igitur tangentialis in corpus, dum elementum M*m* percurrit, alium effectum non exerit, nisi quod motum eius vel acceleret vel retardet" (Vol. I, § 544).

[vii]"In hoc vero eius effectus consistit [...] ut corporis tantum directionem immutet et efficiat, ut corpus, quod per se in recta esset progressurum, in linea curva promoveatur" (Vol.I, § 549).
[viii]"Prima potentia M, cuius directio in superficiem est normalis, nullum habebit effectum in immutando corporis motu, sed tota impendetur in pressionem superficiei. [...]
Secunda potentia N, quia eius directio in ipsa superficie est posita et normalis in directionem corporis, corporis directionem tantum immutabit celeritatem neque augendo neque minuendo. Haec vis igitur corpus a linea brevissima deducet facietque, ut non amplius in plano ad superficiem normali moveatur [...]
Tertia potentia T, quia in directione corporis est posita, celeritatem tantum vel auget vel diminuit" (Vol. II, § 79).
[ix]"Motum enim semel existentem perpetuo conservari debere clare ostendimus supra (§ 63); hic vero, quemadmodum ex motu potentiae oriantur, exposuimus. Quemadmodum vero potentiae sine motu vel existere vel conservari queant, concipi non potest. Quamobrem concludimus omnes potentias, quae in mundo conspiciuntur, a motu provenire" (Vol. II, § 29).
[x]"On appelle en général *puissance* ou *cause motrice*, tout ce qui oblige un Corps à se mouvoir“ (p. 4)
"Cette variation continuelle ne peut provenir (*art.* 6.) que de quelque cause étrangere qui agit sans cesse, pour accélérer ou retarder le Mouvement“ (p. 17).
[xi]"Pourquoi donc aurions-nous recours à ce principe dont tout le monde fait usage aujourd'hui, que la force accélératrice ou retardatrice est proportionnelle à l'élément de la vitesse? principe appuyé sur cet unique axiome vague & obscur, que l'effet est proportionnel à sa cause. [...] nous nous contenterons d'observer, que [...] il est inutile à la Méchanique, & que par conséquent il doit en être banni" (p. xii).
[xii]"Le Mouvement uniforme d'un Corps ne peut être altéré que par quelque cause étrangere. Or de toutes les causes, soit occasionnelles, soit immédiates, qui influent dans le Mouvement des corps, il n'y a tout au plus que l'impulsion seule dont nous soyons en état de déterminer l'effet par la seule connoissance de la cause, comme on le verra dans la seconde Partie de cet Ouvrage. Toutes les autrês causes nous sont entiérement inconnues" (p. 22).
[xv]"La première méthode offre donc beaucoup plus de facilité; aussi est-elle, comme je l'ai observé ci-dessus, presque généralement suivie" (p. xv-xvi).
"La première est presque généralement suivie, comme la plus simple; mais elle a le désavantage d'être fondée sur une notion métaphysique et obscure qui est celle des *forces*" (p. xi-xii).
"j'ai adopté ici la seconde comme je l'avois déjà fait dans la première édition; parce que j'ai voulu éviter la notion métaphysique des forces" (p. xvi).
[xvi]"Ces causes sont-elles la volonté ou la constitution physique de l'homme ou de l'animal qui par son action fait naître le mouvement? Mais qu'est-ce qu'une volonté double ou triple d'une autre volonté, ou une constitution physique capable d'un effet double ou triple d'une autre?" (p. xii).
[xvii]"Si l'on prend le parti de ne point distinguer la cause de l'effet, c'est-à-dire, si l'on entend par le mot *force* la quantité de mouvement même qu'elle fait naître

dans le mobile auquel elle est appliquée, on devient intelligible". (p. xii-xiii) "Je répéterai d'abord, qu'il ne s'agit point ici des causes premières qui font naître le mouvement dans les corps, mais seulement du mouvement déjà produit et inhérent à chacun d'eux. C'est cette quantité de mouvement déjà produite dans un corps, qu'on nomme sa force ou sa puissance" (p. 47). "ainsi que nous l'avons déjà observé, on ne considère, en mécanique, aucune force qui ne réside effectivement dans les corps, c'est-à-dire, qui ne soit réellement une quantité de mouvement déjà produite" (p. 108).

[xix]"*La masse d'un corps est le rapport de deux nombres exprimant combien de fois ce corps et un autre corps choisi arbitrairement et constamment le même, contiennent de parties qui, étant séparées et heurtées deux à deux l'une contre l'autre se communiquent, par le choc, des vitesses opposées égales*" (§ 81).

[xx]"Mais on peut, en général, se dispenser de ces mesurages de vitesse et d'accélération, qui sont délicates et difficiles, et estimer promptement les masses [...] par le *pesage*" (§ 88).

"*Les poids des corps sont, comme l'on voit, en un même lieu, proportionells aux masses*" (§ 89).

[xxi]"*La force ou l'action attractive ou répulsive d'un corps sur un autre est une ligne ayant pour grandeur le produit de la masse de celui ci par l'accélération moyenne de ses points vers ceux du premier et pour direction celle de cette accélération*" (§ 81).

[xxii]See §§ 83, 85, 86, 93, 97, 98, 100. Some examples:

- "l'accélération g qu'ils [les poids] donnent aux masses sur lesquelles ils agissent" (§ 93);
- "Si les forces agissant sur le systême se font équilibre [...]" (§ 119).

[xxiv]"La seule et véritable idée que nous devions nous faire de la force, c'est celle que nous acquérons quand, à l'aide de nos organes, nous cherchons à modifier l'état de repos ou de mouvement des corps qui nous environnent.

Nous éprouvons alors des sensations qui éveillent en nous plusieurs idées fondamentales: d'abord celle de l'existence des corps, puis celle de la forme des corps et des propriétés de l'espace, puis celle du mouvement et du temps, puis encore celle d'une certaine quantité que nous nommons une *pression* ou une *traction*.

Cette quantité est une cause de mouvement ou plutôt une cause de changement de mouvement pour les parties des corps que nous rencontrons à l'aide de nos organes" (p. 37).

[xxv]"Par le mot *force*, on ne doit entendre que les *pressions* ou *tractions* que nous pouvons faire à l'aide de nous organes, sur les corps qui nos environnent" (p. 57).

[xxvi]"La direction de la force sera celle du fil dans lequel elle résidera, et l'intensité de la force dépendra de l'allongement ainsi que de la nature du fil" (p. 46).

[xxvii]"Man pflegt die Mechanik als die Wissenschaft von den *Kräften* zu definiren, und die Kräfte als die *Ursachen*, welche Bewegungen hervorbringen oder hervorzubringen *streben*. Gewiss ist diese Definition [...] Aber ihr haftet die Unklarheit an, von der die Begriffe der Ursache und des Strebens sich nicht befreien lassen. Diese Unklarheit hat sich z. B. gezeigt in der Verschiedenheit

der Ansichten darüber, ob der Satz von der Trägheit und der Satz vom Parallelogramm der Kräfte anzusehen sind als Resultate der Erfahrung, als Axiome oder als Sätze, die logisch bewiesen werden können und bewiesen werden müssen" (p. V).
[xxviii]"Aus diesem Grunde stelle ich es als die Aufgabe der Mechanik hin, die in der Natur vor sich gehenden Bewegungen zu *beschreiben*, und zwar vollständig und auf die einfachste Weise zu beschreiben" (p. V).
[xxix]"Zur Auffassung einer Bewegung sind die Vorstellungen von Raum, Zeit und Materie nöthig, aber auch hinreichend. Mit diesen Mitteln muss die Mechanik suchen, ihr Ziel zu erreichen, und mit ihnen muss sie die Hülfsbegriffe construiren, die sie dabei nöthig hat, z. B. die Begriffe der Kraft und der Masse" (p. 1).
[xxx]"Es ist einleuchtend, dass, wenn man eine bestimmte Bewegung eines Punktes als bedingt durch mehrere Kräfte ansieht, diese nicht einzeln bestimmt sind; nur die Resultante ist bestimmt [...] Aus der Bewegung allein kann die Mechanik nach unserer Auffassung die Definitionen der Begriffe schöpfen, mit denen sie es zu thun hat. Es folgt daraus, dass nach Einführung von Kräftesystemen an Stelle einfacher Kräfte die Mechanik ausser Stande ist, eine vollständige Definition des Begriffs der Kraft zu geben" (p. 11).
[xxxi] See for instance, p. 8, 13, 22, 23, 25, 30, 31, 33, 34, 35.
[xxxii]"Die Definition [der Masse] berücksichtigt lediglich die Tatsache, daß in Wechselbeziehung stehende Körper, ob sogenannte Fernwirkungen, starre oder elastische Verbindungen in Betracht kommen, aneinander Geschwindigkeitsänderungen (Beschleunigungen) bestimmen. Mehr als dies braucht man nicht zu wissen, um mit voller Sicherheit und ohne Furcht, auf Sand zu bauen, definieren zu können" (1933, p. 261).
[xxxiii]"Ist uns aber einmal durch mechanische Erfahrung die Existenz eines besondern beschleunigungbestimmenden Merkmals der Körper nahegelegt, so steht nichts im Wege, willkürlich festzusetzen:
Körper von gleicher Masse nennen wir solche, welche aufeinander wirkend sich gleiche entgegengesetzte Beschleunigungen erteilen. Hiermit haben wir nur ein tatsächliches Verhältnis *benannt*. Analog werden wir in dem allgemeinern Fall verfahren. Die Körper A und B ([...]) erhalten bei ihrer Gegenwirkung beziehungsweise die Beschleunigungen -und +, wobei wir den Sinn derselben durch das Zeichen ersichtlich machen. Dann sagen wir, B hat die -/fache Masse von A. *Nehmen wir den Vergleichskörper A als Einheit an, so schreiben wir jenem Körper die Masse m zu, welcher A das mfache der Beschleunigung erteilt, die er in Gegenwirkung von A erhält*. Das Massenverhältnis ist das negative umgekehrte Verhältnis der Gegenbeschleunigungen" (1933, p. 211-212).
[xxxiv]"Bewegende Kraft ist das Produkt aus dem Massenwert eines Körpers in die an demselben bestimmte Beschleunigung" (1933, p. 242).
[xxxv]"Die Kraft ist also ein bewegungbestimmender Umstand dessen Merkmale sich in folgender Art angeben lassen. Die Richtung der Kraft ist die Richtung der von der gegebenen Kraft allein bestimmten Bewegung. Der Angriffspunkt ist derjenige Punkt, dessen Bewegung auch unabhängig von seinen Verbindungen bestimmt ist. Die Größe der Kraft ist das Gewicht, welches, nach der bestimmten Richtung (an einer Schnur) wirkend, an dem

gegebenen Punkt angreifend, dieselbe Bewegung bestimmt oder dasselbe Gleichgewicht erhält" (1933, p. 75).
[xxxvi]"Durch Anwendung einer jeden dieser drei Methoden können auch die Kräfte aus Rechnungsgrößen zu Gegenständen der unmittelbaren Erfahrung gemacht werden, d.h. zu Zeichen für bestimmte Verbindungen sinnlicher Empfindungen und Wahrnehmungen" (§ 541).
[xxxvii] "Quand on dit que la force est la cause d'un mouvement, on fait de la métaphysique, et cette définition, si on devait s'en contenter, serait absolument stérile. Pour qu'une définition puisse servir à quelque chose, il faut qu'elle nous apprenne à *mesurer* la force; cela suffit d'ailleurs, il n'est nullement nécessaire qu'elle nous apprenne ce que c'est que la force *en soi*, ni si elle est la cause ou l'effet du mouvement" (1897, p. 734).
[xxxviii] "nous devons conclure, qu'avec le système classique, *il est impossible de donner de la force et de la masse une idée satisfaisante*" (1897, p. 736).
[xxxix]"Wir bezeichnen also nun ganz allgemein bei jeder beliebigen Bewegung die Ursache der Bewegung als Kraft und setzen ihre Größe proportional der durch sie bewirkten Beschleunigung. Dieselbe entspricht derjenigen Anstrengung, die wir verspüren würden, wenn wir die nämliche Bewegung, anstatt durch den betreffenden Körper, durch unsere Muskeln hervorrufen würden“ (p. 10).
[xl]"Der physikalische Begriff der Kraft läßt sich nur indirekt durch seine Wirkungen definieren. Wollen wir den **Bewegungszustand oder die Gestalt eines Körpers** z.B. durch Einsatz unserer Muskeln **ändern**, so bedarf es einer Anstrengung, die um so größer ist, je größer die zeitliche Geschwindigkeitsänderung (Beschleunigung) oder je stärker die Deformation sein soll. Diese Anstrengung heißt **Kraft**. [...] Nun beobachten wir überall in unserer Umgebung Änderungen in den Bewegungszuständen gewisser Körper, ohne daß unsere Muskeln direkten Einfluß hätten. Ihre Ursache sehen wir ebenfalls in **Kräften**, welche in gleicher Weise wie unsere Muskeln auf die Körper einwirken" (p. 109).
[xli]"Certains esprits méprisent cette idée vulgaire de la force, comme ils méprisent d'ailleurs la notion de l'effort musculaire.
Ce mépris ne me paraît pas justifié, car seule, la notion vulgaire de la force est la notion féconde; la mécanique, avouons-le hautement, est essentiellement *anthropomorphique*" (p. 138).
[xliii]"L'Anthropomorphisme a joué un rôle historique considérable dans la genèse de la Mécanique; peut-être fournira-t-il encore quelquefois un symbol qui paraîtra commode à quelques esprits; mais il ne peut rein fonder qui ait un caractère vraiment scientifique, ou un caractère vraiment philosophique" (1900, p. 468).
[xliv]"Das 2. Newtonsche Axiom beinhaltet folgende Definitionen und Aussagen:
1. Definition der Masse.
2. Definition der Kraft.
3. [...]" (p. 13-14).
[xlv]"Die Kraft selbst aber definieren wir *nicht* als Ursache der Bewegung; denn die Kraft ist ein Gedankending und keine Naturerscheinung“ (p. 56).

[xlvi]"En réalité la force ([F=ma]) n'est qu'une conception humaine et la cause profonde des mouvements nous est inconnue“ (p. 112).
[xlvii]"Der physikalische Begriff der Kraft beschreibt eben *nicht* etwas unmittelbar Feststellbares [...] Der Kraftbegriff gehört *nicht* zur Formulierung der Abbildungsprinzipien, die etwas im Realtext, d. h. an der Wirklichkeit ([...]) Ablesbares in eine mathematische Form umzuschreiben gestatten“ (p. 145).
[xlviii] “Die Bewegungen und die Beschleunigungen sind Thatsachen, welche beobachtet werden können [...] Wenn man dagegen von Kräften spricht als den Ursachen dieser Bewegungserscheinungen, so weiß man von deren Wesen nichts weiter, als was man eben aus der Beobachtung des Bewegungsvorganges herauslesen kann [...] Man kann daher von der Kraft nichts aussagen, was man nicht bereits von der Beschleunigung weiss“ (p. 24)
[xlix] “Das Trägheitsgesetz sagt aus, dass ein Körper weder eine positive noch eine negative Beschleunigung erfährt, wenn keine äußere Krafteinwirkung vorhanden ist. Beschleunigung ist also immer ein Anzeichen für das Vorhandensein einer solchen äußeren Einwirkung, und zwar das einzige, das die Mechanik kennt“ (p. 114).

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